Formalization of Mathematics

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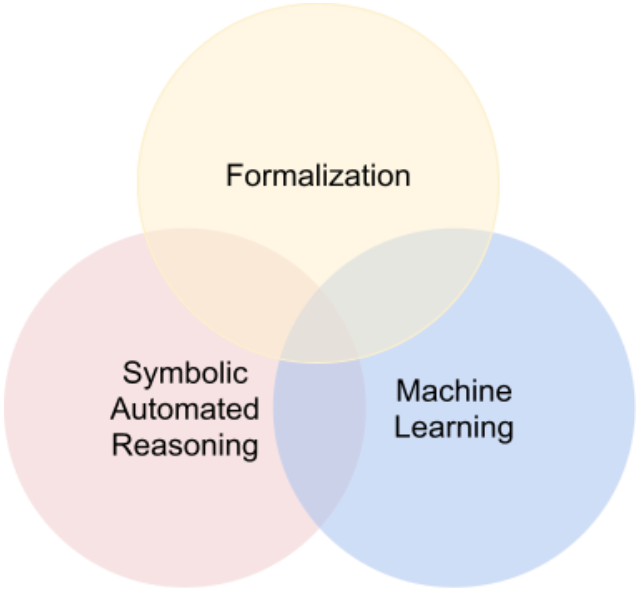
Overview

The landscape

Three components of AI for Mathematics:

- interactive theorem proving and formalization
- automated reasoning and symbolic AI
- machine learning and neural AI

The first two are symbolic methods, in the tradition of formal logic.



The landscape

Two traditions in AI:

- symbolic AI and formal methods
- machine learning and neural networks.

The strengths are complementary:

- symbolic AI is good at getting details right but gets lost.
- machine learning is good at synthesizing data but we don't know what the results mean.

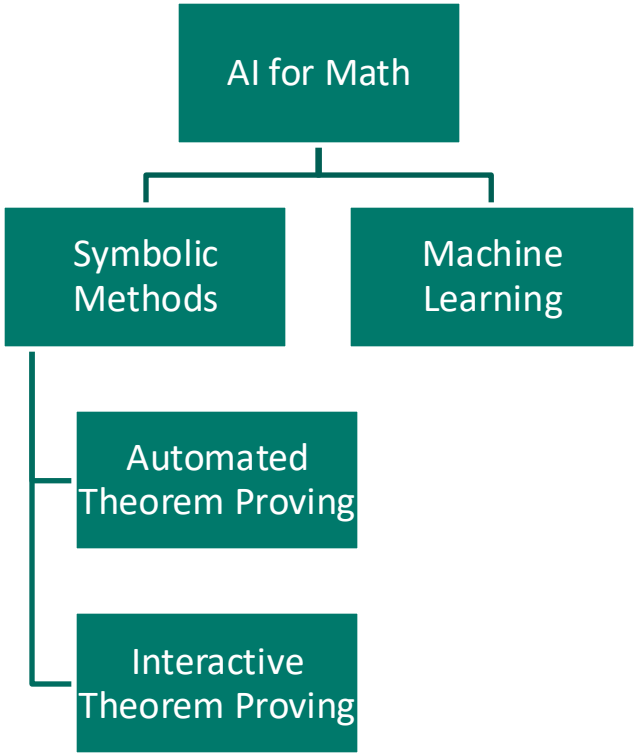
Both are important for mathematics.

The landscape

Historically, automated theorem proving predates interactive theorem proving.

In the 1950s, and early 1960s, researchers began implementing decision procedures and search procedures.

The first interactive proof assistants appeared in the late 1960s.



Formalization of mathematics

In the late 19th and early 20th century, logicians studied formal foundations:

- Gottlob Frege, *Grundgesetze der Arithmetik* (vol. 1, 1893, vol. 2, 1903)
- Guiseppe Peano, *Formulario* (1891 onward)
- Bertrand Russell and A. N. Whitehead, *Principia Mathematica* (1910-1913)
- Hilbert, Ackermann, Bernays, and others

By the early 20th century, it was clear that mathematics can be formalized:

- Statements can be expressed in formal languages, with precise grammar.
- Theorems can be proved from formal axioms, using prescribed rules of inference.

Formalization of mathematics

“The development of mathematics toward greater precision has led, as is well known, to the formalization of large tracts of it, so one can prove any theorem using nothing but a few mechanical rules. The most comprehensive formal systems that have been set up hitherto are the system of Principia Mathematica on the one hand and the Zermelo-Fraenkel axiom system of set theory... on the other. **These two systems are so comprehensive that in them all methods of proof used today in mathematics are formalized, that is, reduced to a few axioms and rules of inference.** One might therefore conjecture that these axioms and rules of inference are sufficient to decide any mathematical question that can at all be formally expressed in these systems.” (Gödel, 1931; my emphasis)

Formalization of mathematics

With the help of computational proof assistants, mathematics is now formalizable *in practice*.

Working with such a proof assistant, users construct a formal axiomatic proof.

In many systems, the proof can be exported and checked independently.

The technology is commonly used for verification of hardware, software, network protocols, cryptographic protocols, cyber-physical systems, and more.

The landscape

Some proof assistants for mathematics:

- Mizar (1973, set theory)
- Isabelle (1986, simple type theory)
- Rocq (1989, dependent type theory)
- HOL Light (1994, simply type theory)
- Lean (2013, dependent type theory)

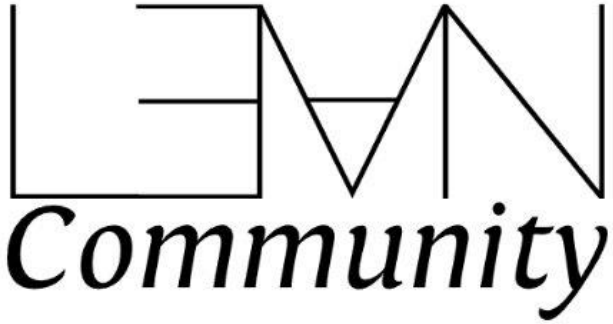
There are many others. See John Harrison, Josef Urban, and Freek Wiedijk, History of Interactive Theorem Proving.

Lean and Mathlib

Leonardo de Moura launched the Lean project in 2013.

Mario Carneiro and Johannes Hölzl split off Mathlib in 2017.

- Mathlib now has almost 2 million lines of code.
- The Lean Zulip channel more than 12,000 members, about 850 active in any two-week period, and almost 2 million messages to date.
- There are have been several celebrated successes.
- There are interesting collaborative projects.
- There have been several articles in the general press.
- There are several meetings and workshops related to Lean.
- There is growing interest and enthusiasm in the mathematical community.



Lean and its Mathematical Library

The Lean theorem prover is a proof assistant developed principally by Leonardo de Moura.

The community recently switched from using Lean 3 to using Lean 4. This website is still being updated, and some pages have outdated information about Lean 3 (these pages are marked with a prominent banner). The old Lean 3 community website has been archived.

The Lean mathematical library, *mathlib*, is a community-driven effort to build a unified library of mathematics formalized in the Lean proof assistant. The library also contains definitions useful for programming. This project is very active, with many regular contributors and daily activity.

You can get a bird's-eye view of what is in the mathlib library by reading the library overview, and read about recent additions on our blog. The design and community organization of mathlib are described in the 2020 article The Lean mathematical library, although the library has grown by an order of magnitude since that article appeared. You can also have a look at our repository statistics to see how the library grows and who contributes to it.

Try it!

You can try Lean in your web browser, install it in an isolated folder, or go for the full install. Lean is free, open source

Learn to Lean!

You can learn by playing a game, following tutorials, or reading books.

Meet the community!

Lean has very diverse and active community. It gathers mostly on a Zulip chat and on GitHub. You

Why Formalize?

Why formalize?

We used to think that the selling point for mathematicians was *verification.*

Gowers: “The notion of a proof assistant sounds rather attractive until you find out that it actually involves a lot more work.”

Verification is important, but it is not what we love about mathematics.

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
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https://doi.org/10.1090/bull/1832
Article electronically published on February 15, 2024

MATHEMATICS AND THE FORMAL TURN

JEREMY AVIGAD

ABSTRACT. Since the early twentieth century, it has been understood that mathematical definitions and proofs can be represented in formal systems with precise grammars and rules of use. Building on such foundations, computational proof assistants now make it possible to encode mathematical knowledge in digital form. This article enumerates some of the ways that these and related technologies can help us do mathematics.

Benefits of formalization

- verifying theorems
- correcting mistakes
- gaining insight
- building libraries
- searching for definitions and theorems
- refactoring proofs
- refactoring libraries
- engineering concepts
- communicating
- collaborating
- managing complexity
- managing the literature
- teaching
- improving access
- using mathematical computation
- using automated reasoning
- using machine learning
- supporting a synthesis of machine learning and symbolic AI

The structure sheaf on `ProjectiveSpectrum 𝒜`.

In `Mathlib/AlgebraicGeometry/Topology.lean`, we have given a topology on `ProjectiveSpectrum 𝒜`; in this file we will construct a sheaf on `ProjectiveSpectrum 𝒜`.

Notation

- `R` is a commutative semiring;
- `A` is a commutative ring and an `R`-algebra;
- `𝒜 : ℕ → Submodule R A` is the grading of `A`;
- `U` is opposite object of some open subset of `ProjectiveSpectrum.top`.

Main definitions and results

We define the structure sheaf as the subsheaf of all dependent function `f : Π x : U, HomogeneousLocalization 𝒜 x` such that `f` is locally expressible as ratio of two elements of the *same grading*, i.e. `∀ y ∈ U, ∃ (V ⊆ U) (i : ℕ) (a b ∈ 𝒜 i), ∀ z ∈ V, f z = a / b`.

- `AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.isLocallyFraction`: the predicate that a dependent function is locally expressible as a ratio of two elements of the same grading.
- `AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.sectionsSubring`: the dependent functions satisfying the above local property forms a subring of all dependent functions `Π x : U, HomogeneousLocalization 𝒜 x`.
- `AlgebraicGeometry.Proj.StructureSheaf`: the sheaf with `U ↦ sectionsSubring U` and natural restriction map.

Then we establish that `Proj 𝒜` is a `LocallyRingedSpace`:

- `AlgebraicGeometry.Proj.stalkIso'`: for any `x : ProjectiveSpectrum 𝒜`, the stalk of

return to top

source

▸ Imports
▸ Imported by

AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.IsFraction
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.isFractionPrelocal
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.isLocallyFraction
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.SectionSubring.zero_mem'
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.SectionSubring.one_mem'
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.SectionSubring.add_mem'
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.SectionSubring.neg_mem'
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.SectionSubring.mul_mem'
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.sectionsSubring
AlgebraicGeometry.ProjectiveSpectrum.StructureSheaf.

LeanSearch

Find theorems in Mathlib4 using natural language query

Query Name or description of the theorem or definition you are looking for

the intermediate value theorem

Number of results 50

Clear | Query Augmentation | Search

Tip: **Query Augmentation** augments your query to increase the chance to find relevant results.

intermediate_value_Icc theorem

```
∀ {α : Type u} [inst : ConditionallyCompleteLinearOrder α] [inst_1 : TopologicalSpace α]
[inst_2 : OrderTopology α]  [inst_3 : DenselyOrdered α] {δ : Type u_1} [inst_4 : LinearOrder
δ] [inst_5 : TopologicalSpace δ]  [inst_6 : OrderClosedTopology δ] {a b : α},  a ≤ b → ∀ {f :
α → δ}, ContinuousOn f (Icc a b) → Icc (f a) (f b) ⊆ f '' Icc a b
```

▸ Intermediate Value Theorem for Continuous Functions on Closed Intervals

Go to doc

intermediate_value_Ioo theorem

```
∀ {α : Type u} [inst : ConditionallyCompleteLinearOrder α] [inst_1 : TopologicalSpace α]
[inst_2 : OrderTopology α]  [inst_3 : DenselyOrdered α] {δ : Type u_1} [inst_4 : LinearOrder
δ] [inst_5 : TopologicalSpace δ]  [inst_6 : OrderClosedTopology δ] {a b : α},  a ≤ b → ∀ {f :
α → δ}, ContinuousOn f (Icc a b) → Ioo (f a) (f b) ⊆ f '' Ioo a b
```

▸ Intermediate Value Theorem for Continuous Functions on Closed Intervals and Open Image Intervals

Go to doc

2 Local theory of convex integration

2.1 Key construction

The goal of this chapter is to explain the local aspects of (Theillière's implementation of) convex integration, the next chapter will cover global aspects.

The elementary step of convex integration modifies the derivative of a map in one direction. The precise meaning of "one direction" relies on the following definition.

Definition 2.1. ✓

> *A dual pair on a vector space E is a pair (π, v) where π is a linear form on E and v a vector in E such that $\pi(v) = 1$.*

Let E and F be finite dimensional real normed vector spaces. Let $f\colon E \to F$ be a smooth map, and let (π, v) be a dual pair on E. We want to modify Df in the direction of v while almost preserving it on $\ker \pi$. Say we wish $Df(x)v$ could live in some open subset $\Omega_x \subset F$. Assume there is a smooth family of loops $\gamma\colon E \times \mathbb{S}^1 \to F$ such that each γ_x takes values in Ω_x, and its average value $\overline{\gamma}_x = \int_{\mathbb{S}^1} \gamma_x$ is $Df(x)v$ for all x. Obviously such loops can exist only if $Df(x)v$ is in the convex hull of Ω_x, and we saw in the previous chapter that this is almost sufficient (and we'll see this is sufficiently almost sufficient for our purposes). Then we can modify f to fulfil our wish using the following construction.

Definition 2.2. (Theillière 2018)✓

> *The map obtained by corrugation of f in direction (π, v) using γ with oscillation number N is*
>
> $$x \mapsto f(x) + \frac{1}{N} \int_0^{N\pi(x)} [\gamma_x(s) - \overline{\gamma}_x] ds.$$

In the above definition, we mostly think of N as a large natural number. But we don't actually require it, any positive real number will do.

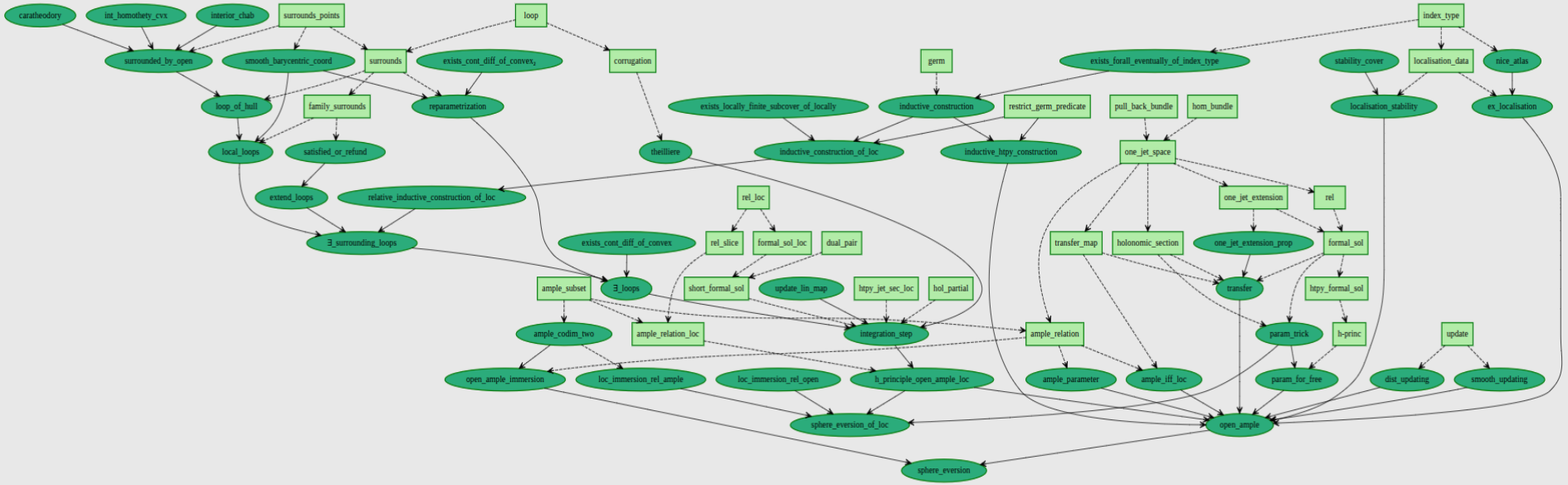
The next proposition implies that, provided N is large enough, we have achieved $Df'(x)v \in \Omega_x$, almost without modifying derivatives in the directions of $\ker \pi$, and almost without moving $f(x)$.

Proposition 2.3. (Theillière 2018)✓

> *Let f be a $\mathcal{C}^1$ function from E to F. Let (π, v) be a dual pair on E. Let $\gamma\colon E \times \mathbb{S}^1 \to F$*

Dependencies

Legend



Welcome to the Natural Number Game

An introduction to mathematical proof.

In this game, we will build the basic theory of the natural numbers `{0,1,2,3,4,...}` from scratch. Our first goal is to prove that `2 + 2 = 4`. Next we'll prove that `x + y = y + x`. And at the end we'll see if we can prove Fermat's Last Theorem. We'll do this by solving levels of a computer puzzle game called Lean.

Read this.

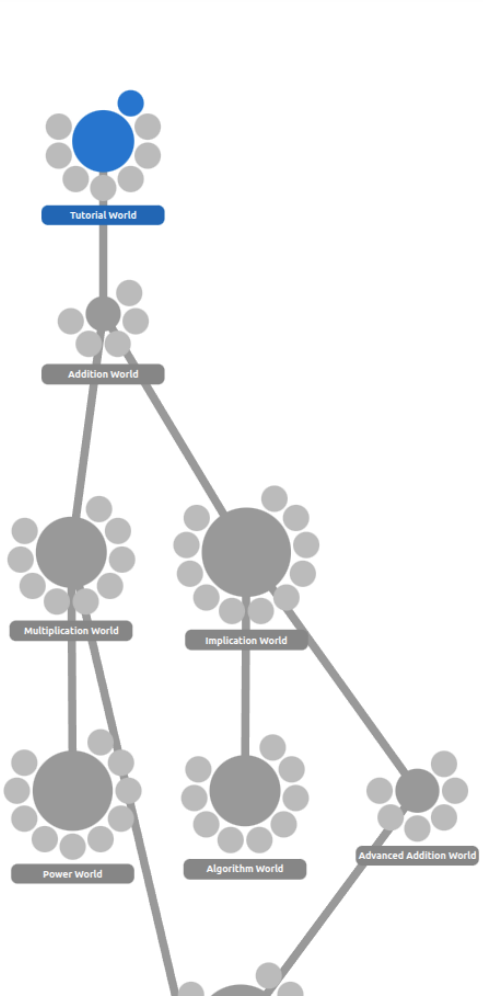
Learning how to use an interactive theorem prover takes time. Tests show that the people who get the most out of this game are those who read the help texts like this one.

To start, click on "Tutorial World".

Note: this is a new Lean 4 version of the game containing several worlds which were not present in the old Lean 3 version. More new worlds such as Strong Induction World, Even/Odd World and Prime Number World are in development; if you want to see their state or even help out, checkout out the issues in the github repo.

More

Click on the three lines in the top right and select "Game Info" for resources, links, and ways to interact with the Lean community.



Theorems | Tactics | Definitions

apply cases contrapose
decide exact have induction
intro left rfl right rw
simp simp_add symm tauto
trivial use

1. Proofs by calculation

This book begins in the familiar world of numbers: $\mathbb{N}$, the natural numbers (which in this book include 0); $\mathbb{Z}$, the integers; $\mathbb{Q}$, the rational numbers; and $\mathbb{R}$, the real numbers. We solve problems which feel pretty close to high school algebra – deducing equalities/inequalities from other equalities/inequalities – using a technique which is not usually taught in high school algebra: building a single chain of expressions connecting the left-hand side with the right.

1.1. Proving equalities

1.1.1. Example

We start with proofs of equalities. Here is a typical example of the technique mentioned.

Problem

Let a and b be rational numbers and suppose that $a - b = 4$ and $ab = 1$. Show that $(a + b)^2 = 20$.

Solution

$$\begin{aligned}(a+b)^2 &= (a-b)^2 + 4ab \\ &= 4^2 + 4 \cdot 1 \\ &= 20.\end{aligned}$$

We call the above proof a *proof by calculation*. The goal was to show that $(a + b)^2 = 20$, and we

From Scholze's second blog post

Question: Did you learn anything about mathematics during the formalization?'

Answer: Yes! The first is a beautiful realization of Johan Commelin. Basically, the computation of the Ext-groups in the Liquid Tensor Experiment is done via a certain non-explicit resolution known as a Breen-Deligne resolution…. The Breen-Deligne resolution has certain beautiful structural properties, but is not explicit, and the existence relies on some facts from stable homotopy theory. In order to formalize Theorem 9.4, the Breen-Deligne resolution was axiomatized, formalizing only the key structural properties used for the proof. What Johan realized is that one can actually give a nice and completely explicit object satisfying those axioms, and this is good enough for all the intended applications. This makes the rest of the proof of the Liquid Tensor Experiment considerably more explicit and more elementary, removing any use of stable homotopy theory. I expect that Commelin's complex may become a standard tool in the coming years.

From Scholze's second blog post

Question: What else did you learn?

Answer: What actually makes the proof work! When I wrote the blog post half a year ago, I did not understand why the argument worked, and why we had to move from the reals to a certain ring of arithmetic Laurent series. But during the formalization, a significant amount of convex geometry had to be formalized (in order to prove a well-known lemma known as Gordan's lemma), and this made me realize that actually the key thing happening is a reduction from a non-convex problem over the reals to a convex problem over the integers.

ABSTRACTION BOUNDARIES AND SPEC DRIVEN DEVELOPMENT IN PURE MATHEMATICS

JOHAN COMMELIN AND ADAM TOPAZ

ABSTRACT. In this article we discuss how *abstraction boundaries* can help tame complexity in mathematical research with the help of an interactive theorem prover. While many of the ideas we present here have been used implicitly by mathematicians for some time, we argue that the use of an interactive theorem prover introduces additional qualitative benefits in the implementation of these ideas.

1. INTRODUCTION

Modern research in pure mathematics has a clear tendency toward increasing complexity. New striking mathematical results may involve complex proof techniques, deep mastery of a subfield within mathematics, or nontrivial input from several areas of mathematics. All of these play a role in increasing the inherent complexity of a piece of modern mathematics.

In many respects, such increasing complexity is an indication of *progress* in pure mathematics. However, with such complexity comes a significant increase in *cognitive load* for both readers and authors. It is now routine to see significant new papers in pure mathematics with one hundred pages or more. Similarly, the refereeing process of a complex mathematical result regularly takes multiple years.

Milestones

Milestones

Early milestones:

- the prime number theorem (2004, A, Donnelly, Gray, and Raff, in Isabelle)
- the four-color theorem (2004, Gonthier and Werner, in Rocq)
- Jordan Curve theorem (2005, Hales, In HOL Light)

Bigger milestones:

- Feit-Thompson theorem (Gonthier and many others, in Rocq, 2012)
- Flyspeck project (Hales and many others, In HOL Light and Isabelle, 2014)

Milestones

Moving toward contemporary mathematics:

- Perfectoid spaces
- The liquid tensor experiment
- The sphere eversion project
- On a density conjecture about unit fractions
- The polynomial Freiman-Ruzsa conjecture
- Exponentially improved upper bounds on Ramsey’s theorem
- Carleson’s theorem
- The resolution of the Aharoni-Korman conjecture
- Fermat's last theorem

Thomas Bloom's result on unit fractions

Theorem. Any subset A of $\mathbb{N}$ of positive upper density contains a finite subset $S \subset \mathbb{N}$ satisfying $\sum_{n \in S} \frac{1}{n} = 1$.

Kevin Buzzard
@XenaProject

Happy to report that Bloom went on to learn Lean this year and, together with Bhavik Mehta, has now formalised his proof in Lean b-mehta.github.io/unit-fractions/ (including formalising the Hardy-Littlewood circle method), finishing before he got a referee's report for the paper ;-)

Timothy Gowers @wtgowers
@wtgowers

Very excited that Thomas Bloom and Bhavik Mehta have done this. I think it's the first time that a serious contemporary result in "mainstream" mathematics doesn't have to be checked by a referee, because it has been checked formally. Maybe the sign of things to come ... 1/ t.co/Ue7n9RuaF2

This post is unavailable.

5:12 AM · Jun 13, 2022

The Polynomial Freiman-Ruzsa Conjecture

On November 9, 2023, W. T. Gowers, Ben Green, Freddie Manners, and Terence Tao posted a proof of the following on arXiv.

Theorem. Suppose that $A \subset F_2^n$ is a set with $|A + A| \leq K|A|$. Then A is covered by at most $2K^C$ cosets of some subgroup $H \leq F_2^n$ of size at most $|A|$.

Tao enlisted a team of people to help formalize it.

The achievement was featured in an article in *Quanta* on December 6.

The Polynomial Freiman-Ruzsa Conjecture

A digitisation of the proof of the Polynomial Freiman-Ruzsa Conjecture in Lean 4

Blueprint | Documentation | Paper | View on GitHub

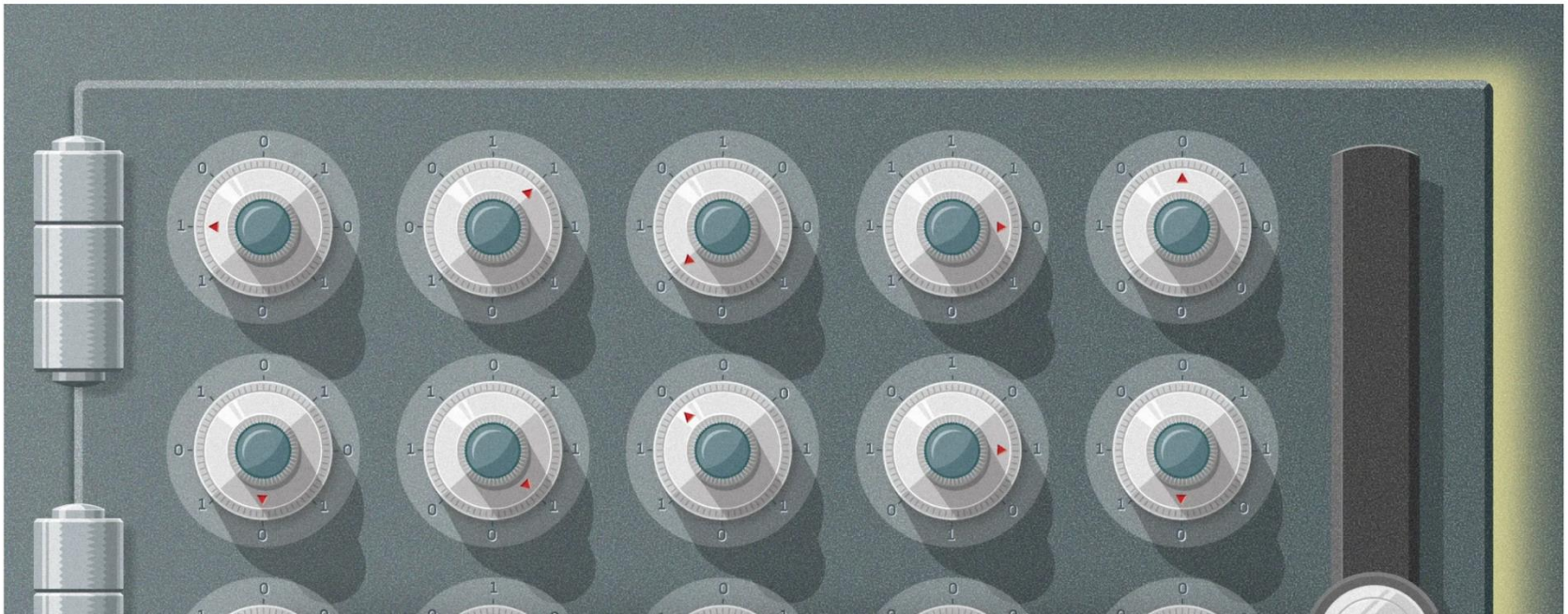
The Polynomial Freiman-Ruzsa Conjecture

The purpose of this repository is to hold a Lean4 formalization of the proof of the Polynomial Freiman-Ruzsa (PFR) conjecture (see also this blog post). The statement is as follows: if A is a non-empty subset of $\mathbf{F}_2^n$ such that $|A+A| \leq K|A|$, then A can be covered by at most $2K^{12}$ cosets of a subspace H of $\mathbf{F}_2^n$ of cardinality at most $|A|$. The proof relies on the theory of Shannon entropy, so

COMBINATORICS

‘A-Team’ of Math Proves a Critical Link Between Addition and Sets

A team of four prominent mathematicians, including two Fields medalists, proved a conjecture described as a “holy grail of additive combinatorics.” Within a month, a loose collaboration verified it with a computer-assisted proof.



Carleson's Theorem

Theorem. Let f be an L^p periodic function, $p \geq 1$, with Fourier coefficient $\hat{f}(n)$. Then

$$\lim_{N\to\infty} \sum_{|n|\leq \mathbb{N}} \hat{f}(n) e^{inx} = f(x)$$

for almost every x.

Christophe Thiele and his group in Bonn proved a generalization. It was written as a Lean blueprint (144 pages) and verified by a team led by Floris van Doorn.

Carleson operators on doubling metric measure spaces

A formalization in Lean 4

Blueprint (html) | Blueprint (pdf) | Formalization | View on Github

Formalization of a generalized Carleson's theorem

A (WIP) formalized proof of a generalized Carleson's theorem in Lean.

- Zulip channel for coordination
- Blueprint
- Blueprint as pdf
- Dependency graph

A result in combinatorics

Lawrence Hollom recently refuted a 1992 conjecture on posets satisfying the Finite Antichain Condition.

Bhavik Mehta quickly verified (and corrected) the construction in Lean.

Mathematics > Combinatorics

[Submitted on 25 Nov 2024 (v1), last revised 22 May 2025 (this version, v4)]

A resolution of the Aharoni-Korman conjecture

Lawrence Hollom

A poset P is said to satisfy the finite antichain condition, or FAC for short, if it has no infinite antichain. It was conjectured by Aharoni and Korman in 1992 that any FAC poset P possesses a chain C and a partition into antichains such that C meets every antichain of the partition. Our main results are twofold. We provide a counterexample to the conjecture in full generality, but, despite this, we also prove that the conjecture does hold true for a broad class of posets. In particular, we prove that the Aharoni-Korman conjecture holds for countable posets avoiding intervals I such that either I or its reverse I^* is of the form $\bigoplus_{x\in\omega} \mathrm{Q}_x$, where each Q_x is infinite and co-wellfounded.

In pursuit of these goals, we also investigate other facets of the structure of FAC posets. In particular, we consider strongly maximal chains in FAC posets, proving some results, and posing several questions and conjectures.

7. Acknowledgements

The author would like to thank Béla Bollobás for his thorough reading of the manuscript and many valuable comments. The author would also like to thank

[redacted]

Bhavik Mehta for producing a formal verification of Proposition 5.7 (available at [19]), and for helping find and resolve several inaccuracies in the original proof of this statement. Thanks are also due to Nikolai Beluhov and George Bergman for each pointing out several inaccuracies in the manuscript, and for further suggestions which significantly improved the presentation of the paper. The author is funded by the internal graduate studentship of Trinity College, Cambridge.

A result in theoretical computer science

Noah Singer and Ryan O'Donnell recent proved results on expanders relying on lengthy group-theoretic computations.

Singer verified a key construction with another PhD student and two undergraduate students at Carnegie Mellon.

Coboundary expansion inside Chevalley coset complex HDXs

Ryan O'Donnell*[ⓡ] Noah G. Singer†[ⓡ]

Abstract

Recent major results in property testing [BLM24, DDL24] and PCPs [BMV24] were unlocked by moving to high-dimensional expanders (HDXs) constructed from $\widetilde{C}_d$-type buildings, rather than the long-known $\widetilde{A}_d$-type ones. At the same time, these building quotient HDXs are not as easy to understand as the more elementary (and more symmetric/explicit) *coset complex* HDXs constructed by Kaufman–Oppenheim [KO18] (of A_d-type) and O'Donnell–Pratt [OP22] (of B_d-, C_d-, D_d-type). Motivated by these considerations, we study the B_3-type generalization of a recent work of Kaufman–Oppenheim [KO21], which showed that the A_3-type coset complex HDXs have good 1-coboundary expansion in their links, and thus yield 2-dimensional topological expanders.

The crux of Kaufman–Oppenheim's proof of 1-coboundary expansion was: (1) identifying a group-theoretic result by Biss and Dasgupta [BD01] on small presentations for the A_3-unipotent group over $\mathbb{F}_q$; (2) "lifting" it to an analogous result for an A_3-unipotent group over polynomial extensions $\mathbb{F}_q[x]$.

For our B_3-type generalization, the analogue of (1) appears to not hold. We manage to circumvent this with a significantly more involved strategy: (1) getting a computer-assisted proof of vanishing 1-cohomology of B_3-type unipotent groups over $\mathbb{F}_5$; (2) developing significant new "lifting" technology to deduce the required quantitative 1-cohomology results in B_3-type unipotent groups over $\mathbb{F}_{5^k}[x]$.

Algebra Is Half the Battle: Verifying Presentations of Graded Unipotent Chevalley Groups

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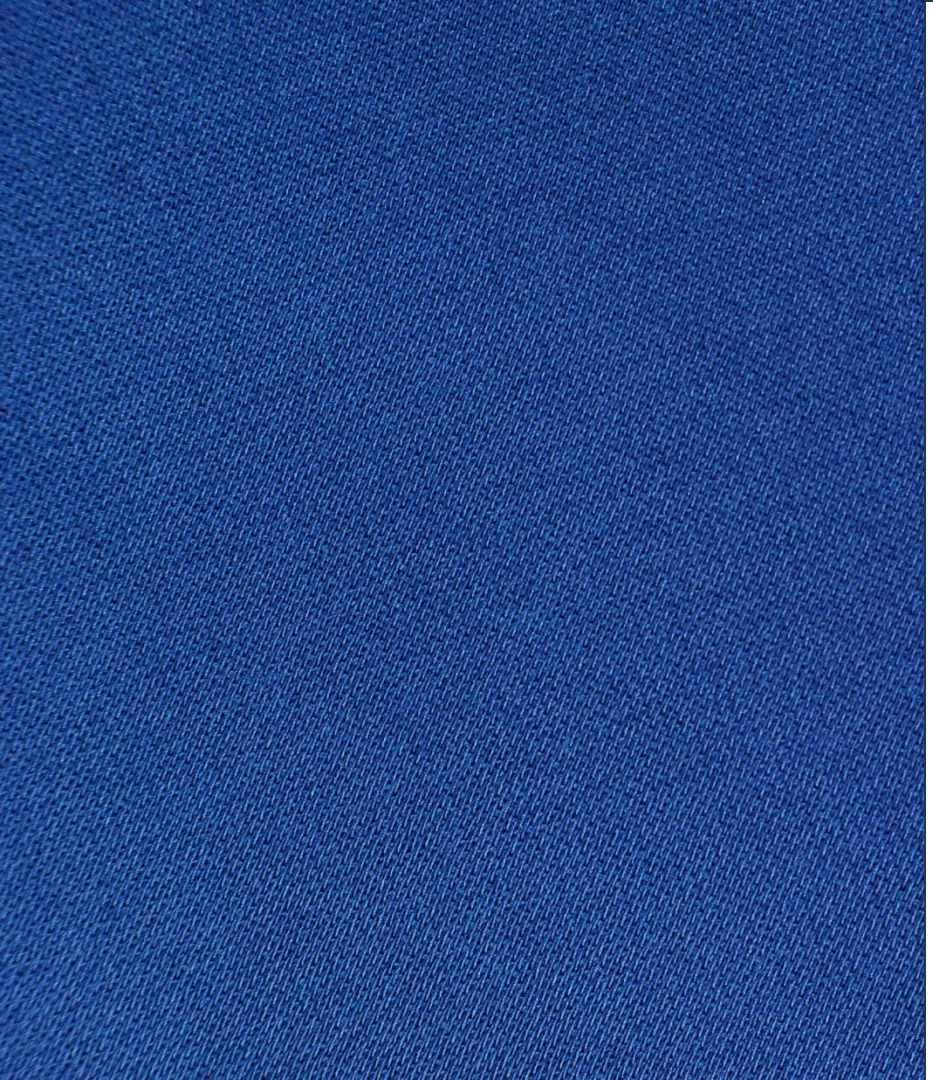
Cayden Codel
School of Computer Science, Carnegie Mellon University, Pittsburgh, PA, USA

Noah G. Singer
School of Computer Science, Carnegie Mellon University, Pittsburgh, PA, USA

Abstract

Graded unipotent Chevalley groups are an important family of groups on matrices with polynomial entries over a finite field. Using the Lean theorem prover, we verify that three such groups, namely, the A_3- and the two B_3-type groups, satisfy a useful group-theoretic condition. Specifically, these groups are defined by a set of equations called Steinberg relations, and we prove that a certain canonical "smaller" set of Steinberg relations suffices to derive the rest.

Our work is motivated by an application for building topologically-interesting objects called higher-dimensional expanders (HDXs). In the past decade, HDXs have formed the basis for many new results in theoretical computer science, such as in quantum error correction and in property testing. Yet despite the increasing prevalence of HDXs, only two methods of constructing them are known. One such method builds an HDX from groups that satisfy the aforementioned property, and the Chevalley groups we use are (essentially) the only ones currently known to satisfy it.



Trying it out

Trying it out

- Start with the Lean community pages.
- See, in particular, the learning resources and the Natural Number Game.
- You can also browse Mathlib.
- Check out the Lean Zulip social media platform.
- You can easily find the projects we discussed here online.

Conclusions

Conclusions

Like the digitization of language, the digitization of mathematics has many uses:

- communication
- collaboration
- search
- verification
- exploration
- uses of automation and AI.

The precise nature of mathematics makes some of these even more compelling:

- Mathematical objects and proofs are complex.
- It’s not mathematics unless the details are right.

Conclusions

The community of formalizers is small but growing.

Points of access:

- the Lean community web pages
- the Lean Zulip channel
- the Institute for Computer-Aided Reasoning in Mathematics.

Some people like working with a proof assistant, some people don’t.

Be open to opportunities and collaboration.