Working Paper

# Strategic Information Gathering with Possibility of Overlaps

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# STRATEGIC INFORMATION GATHERING WITH POSSIBILITY OF OVERLAPS[1]

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In this paper we provide a novel approach to modeling information acquisition. The approach allows to account explicitly for the correlation of information coming from various sources and the possibility of overlaps in learning. As a possible application of the approach, we study the Kyle-type trading model and show that overlaps in doing market research have significant entry-deterrence effect and can explain the oligopolistic structure of the industry, as well as cross-sectional variation in analyst coverage for different stocks.

KEYWORDS: asymmetric information, information acquisition, insider trading, informational barriers to entry.

## 1. INTRODUCTION

In this paper we introduce a way of modeling information acquisition by multiple agents. Unlike the existing literature where information is represented as a noisy signal of the unknown truth (Verrecchia [1982] and others), we explicitly model information as coming from various sources, we allow agents to overlap by exploring the same sources of information, and we account for varying degree of severity of these overlaps, depending on varying research environments. The approach used in the literature is equivalent to ours for single-agent models, however it requires pretty restrictive assumptions[1] once we consider multi-agent models with possibly asymmetric behavior in equilibrium. In this paper we develop a more flexible approach, which at the same time represents a more natural way of thinking about

[1] I appreciate discussions with Burton Hollifield, Richard Green, Jack Stecher, Yaroslav Kryukov. Errors are mine alone.

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[1] Such as independence of noise across agents. These assumptions have important implications, while it is questionable whether they constitute a natural way of thinking about research in real world situations.

information acquisition — as exploring wide range of sources and combining multiple bits of information. It naturally accounts for various sources of such information and its possible correlation, and it allows for overlaps in information when several agents compete in performing this "intelligence gathering".

Our way of modeling learning is along the lines of general "entropy reduction" methodology (relevant references will be here). Our contribution is that we describe the properties of the information obtained when two or more agents compete in doing "entropy reduction" simultaneously.

In various economic settings featuring uncertainty agents have incentives to do "research" and these incentives affect the equilibrium outcomes. In financial sector endogenous research efforts by market participants is an important determinant of trading strategies and market outcomes. Informational advances of one firm relative to others allow for positive expected profits form trading. This may apply to various settings, including but not limited to trading firms learning company's fundamentals from various sources and trading on this information; large banks adding various ingredients to their pricing models of complicated risk-sharing contracts that allow them to price these contracts better than others, and so on. One can think of skill-development process or financial expertise in a similar fashion, given that any enhanced skill results in possession of enhanced information. This process of information-acquisition and skill-development is often referred to as typical market behavior: *"Typical market behavior, such as hedge funds seeking to gain an edge by gathering intelligence on a company from a wide range of sources."*[2] This intelligence-gathering process is the main focus of this paper.

We use this approach to endogenize research efforts in a one-period two-firm trading model in the spirit of Kyle [1985a] and discuss both symmetric

[2]WSJ, Nov 23 2010

and asymmetric Nash equilibria, study their existence and possible multiplicity, and demonstrate entry-deterrence effects of purely informational barriers to entry.

The application of different research technologies to a trading environment yields several interesting results. There is both theoretical and empirical literature studying how number of analysts covering stocks affects market depth (Brennan and Subrahmanyam [1995], Holden and Subrahmanyam [1992] and others). There are papers that try to endogenize the number of analysts, such as Dierker [2006], however in these models entry happens until all analysts get zero profit in equilibrium and thus the market of doing research is assumed to be competitive. As a particular implementation of the general approach to modeling research, in this paper we make a new attempt to endogenize the number of analysts and explain cross-sectional variation in analyst coverage. The contribution of this paper is a rationalization of oligopolistic structure due to purely informational barriers to entry. Our model can produce a situation with one monopolist analyst enjoying significant profits from research and at the same time nobody can enter purely due to structure of informational uncertainty and possibility of overlaps in information. The model can be also used to rationalize possible asymmetries in analysts' expertise. The strength of these barriers will naturally vary across industries, thus explaining the cross-sectional variation in number of analysts covering different stocks. However, insider trading is only one single application of a more general approach to modeling research we discuss, and it can be used to study other economic situations.

The results of this paper can partially explain high levels of specialization in obtaining financial expertise in pricing and trading complex securities and financial contracts (relevant references will be here). We say "partially" because the trading model we use does not feature over-the-counter trading, which is typical for these instruments. However all such contracts require dealers to build a complicated pricing model. Building a pricing model can be decomposed into several steps — starting with Black-Scholes setup, adding inflation effects, exchange-rate effects, possibility of defaults and so on — and this path of adding ingredients to a pricing model needs to be conquered by any dealer who wants an efficient and realistic model. Each model ingredient provides some insight about the fundamental price, thus constitutes a separate source of information. When this process is relatively standard and each dealer has to follow this path more or less in one direction — from a simple model to a very complicated one — it corresponds to a research technology with high severity of overlaps and we have high entry-deterrence results for it. Also there is a way to reframe Kyle-type trading model into an idealized complicated-contract selling model,[3] so our paper gives some hint along these lines.

The structure of the exposition is as follows. In section 2 we develop the general information acquisition approach. In section 3 we discuss the Kyle-type trading environment and use our approach to endogenize research efforts by trading firms (we will refer to them as research firms). In section 4 we discuss the results obtained and provide some numerical examples of the arising equilibria. Section 5 concludes the discussion. Proofs of all propositions are in the Appendix.

[3]The way to reframe Kyle model: insiders = dealers originating contracts, market-maker = competitive strategic traders with elastic demand for any underpriced/overpriced contracts, liquidity traders = corporate entities with exogenous demand for these contracts. Insiders are the dealers who use a pricing model to trade contracts. The market-maker is a competitive fringe of strategic traders who hunt for order-flow information and use this information to price contracts, they don't have any pricing model on their own. In equilibrium you may have dealers with different levels of pricing model sophistication, trading the same type of contracts and getting different expected profits (the poorer the model, the lower the expected profit). If you think about Kyle in this way, it becomes a fairly idealized model of contract origination.

## 2. RESEARCH TECHNOLOGIES

We can think of a prior uncertainty, symmetrically faced by all firms in a model, as a random variable $\tilde{v} \sim N(0, \Sigma_0)$. For our approach we assume Normal distribution for $\tilde{v}$.[4] A research technology may include (but is not limited to) a set of paths that each firm follows in gathering information: one firm may start with analyzing trends in macro-data that provide initial insight into the true value of some asset, then do networking with industry experts, then analyze online subscription databases and so on, while other firms may explore other sources, or the same sources in a different sequence. Formally, a research technology will determine the joint distribution of informative signals each firm obtains.

Definition 2.1 A research technology *is a mapping from firms' research efforts* $\alpha = (\alpha_1 \ldots \alpha_N)$ *to the joint distribution of firms' signals and the uncertain value* $\tilde{v}$*:* $\mathcal{G}(\tilde{v}, s_1 \ldots s_N)$*, where* $s_i$ *is the signal i-th firm obtains,* $i \in \{1 \ldots N\}$*.*

**Example**. Consider the research technology used by Verrecchia [1982]. Each trader observes true return $\tilde{v}$ perturbed by some noise $\tilde{\epsilon}_i$, a random variable which has a normal distribution with mean zero and precision $\sigma_i \geq 0$, and is independent of the perturbations of other traders. In Verrecchia [1982] traders acquire precision $\sigma_i$ at cost, and precision is the inverse of $\tilde{\epsilon}_i$ variance. We can treat precision $\sigma_i$ as a direct result of applying research effort $\alpha_i$, thus $\sigma_i = f(\alpha_i) = \alpha_i$.[5] Then $s_i = \tilde{v} + \tilde{\epsilon}_i$ and:

$$\{\tilde{v}, s_1 \ldots s_N\} \sim N(0, \Omega),$$

[4] Our approach can be extended to models with general distribution for $v$ that can be expressed as a monotonic transformation of a Normally-distributed random variable.

[5] Note that in this model $\alpha_i$ can be any positive real number, $\alpha_i \in \mathbb{R}^+$

$$\text{where } \Omega = \begin{pmatrix} \Sigma_0 & \Sigma_0 & \cdots & \Sigma_0 \\ \Sigma_0 & \Sigma_0 + 1/\alpha_1 & \cdots & \Sigma_0 \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_0 & \Sigma_0 & \cdots & \Sigma_0 + 1/\alpha_N \end{pmatrix}$$

This mapping from a vector of research intensities $\alpha = (\alpha_1, ...\alpha_N)$ to the joint distribution of resulting signals is an example of a research technology. It can be seen that the structure of traders' signals and the independence of noise $\tilde{\epsilon}_i$ across traders implies no effect of traders' research efforts $\alpha_i$ on covariances of signals. Also note that this research technology does not account for variety of sources of information – each trader gathers all available information irrespective of his research effort (common component $\tilde{v}$), and then spends resources to minimize the idiosyncratic noise.

In this paper we propose three alternative ways of modeling information acquisition.

In Verrecchia [1982] traders were obtaining a noisy signal of the true value $\tilde{v}$. Alternatively in this paper we assume there is no noise associated with the learning process, while the overall uncertainty is composed of several bits of information — these bits may come from various sources and are not perfectly correlated. Each research firm learns a subset of them, and the size of this subset depends on costly research effort. We normalize the maximum research effort to 1 (100%), meaning all possible sources of information have been explored and there is no remaining uncertainty faced by the firm. Adding noise as in Verrecchia [1982] is possible but not necessary. The 100% research effort corresponds to an infinitely precise signal in Verrecchia [1982].

Suppose there are $M$ sources of information, each source contributes a random variable $\tilde{e}_i$, $i \in \{1 \dots M\}$ to the overall uncertainty. A firm exploring the $i$-th source learns the realization $e_i$ of random variable $\tilde{e}_i$. The overall uncertainty — the random variable $\tilde{v}$ — is assumed to be the sum of all sources' contributions:

$$\tilde{v} = \sum_{i=1}^{M} \tilde{e}_i \tag{2.1}$$

We will refer to $\tilde{e}_i$ as individual bits of information, and for simplicity we assume they are identically normally distributed. It is natural to think about research effort $\alpha \in [0,1]$ as a proportion of sources explored by a firm. If $M$ is even, $\alpha = 0.5$ would mean the firm is able to pick $M/2$ sources and learn corresponding realizations $e_i$, while the other $M/2$ sources will constitute the remaining uncertainty.

Now we introduce the notion of "internal correlation" to capture the possible correlation of information coming from different sources:

Definition 2.2 *Suppose $M$ is an even number. Pick any $M/2$ individual bits of information and let their sum be a random variable $\tilde{e}_1$, while the sum of the remaining bits denote by $\tilde{e}_2$. Then we will refer to $\hat{\rho} = corr(\tilde{e}_1, \tilde{e}_2)$ as the "internal correlation" of information. We will assume $\hat{\rho} \in [0,1)$.*

Suppose one firm made a research effort $\alpha$ and assume for now that $\alpha M$ is a Natural number. The resulting signal the firm obtains is:

$$\tilde{s}_i = \sum_{j=1}^{\alpha M} \tilde{e}_j \tag{2.2}$$

The variance of this signal and its covariance with $\tilde{v}$ will not depend on which of the $M$ sources in particular the firm was able to explore, due to the identical distribution assumption. Moreover, the joint distribution of $\tilde{s}_i$ and $\tilde{v}$ should not depend on $M$ itself — according to the above setup, knowing 1 piece out of 4 should be the same as knowing 2 pieces out of 8, since we can always split each of the 4 initial pieces into two halves. The latter allows us to exactly identify the moments of the $(\tilde{s}_i, \tilde{v})$ joint normal distribution:

Proposition 2.1 *Suppose a firm with research effort $\alpha \in [0,1]$ obtains a signal $\tilde{s}_i$. Let the internal correlation be $\hat{\rho} \in [0,1)$ and define $\rho = 2\hat{\rho}/(1+\hat{\rho})$.*[6] *Then:*

$$
\begin{aligned}
Var(\tilde{s}_i) &= \alpha\Sigma_0(1-(1-\alpha)\rho) \\
Cov(\tilde{s}_i, \tilde{v}) &= \alpha\Sigma_0
\end{aligned}
\tag{2.3}
$$

*See Appendix A.1 for proof.*

Once there are two or more research firms learning about $\tilde{v}$ in the described fashion, the covariance of their signals will depend on how often these firms overlap. **Example**: Suppose $M = 6$ and there are three firms with research efforts $\alpha_1 = 1/6$, $\alpha_2 = 1/3$, and $\alpha_3 = 1/2$. The first firm is entitled to reveal the realization of one bit of information, the second firm can choose two bits, and the third chooses three. It is possible that neither of the three firms will overlap, as if firm 1 reveals $\{e_1\}$, firm 2 reveals $\{e_2, e_3\}$ and firm 3 — $\{e_4, e_5, e_6\}$, and we will refer to this case as **non-overlapping research**. The other extreme is when all firms reveal informative bits in exactly same sequence, as if firm 1 reveals $\{e_1\}$, firm 2 reveals $\{e_1, e_2\}$ and firm 3 — $\{e_1, e_2, e_3\}$, we will refer to this case as **perfectly-overlapping research**. We will study the third possibility, when each firm makes an equally-likely random draw of informative bits to reveal, we will refer to this case as **randomly-overlapping research**.

Definition 2.3 *Suppose there are $N$ research firms on the market and $\alpha_i$ is the research effort of i-th firm. Suppose for $\forall i \in \{1 \dots N\}$ $\alpha_i M$ is a Natural number. Denote by $S_i$ the subset of $M$ bits of information each firm is entitled to reveal.*

1. *Firms engage in* non-overlapping research *if $\forall i \neq j$ we have $S_i \cap S_j = \varnothing$. This can happen only if $\sum_{i=1}^{N} \alpha_i \leq 1$.*

[6] $\rho$ is a monotonically increasing transformation of $\hat{\rho}$, preserving the domain: $\rho \in [0,1)$

2. *Firms engage in* perfectly-overlapping research *if* $\forall i \neq j$ *such that* $\alpha_i \geq \alpha_j$ *we have* $S_i \cap S_j = S_j$.
3. *Firms engage in* randomly-overlapping research *if* $\forall i$ $S_i$ *is a simple random sample of size* $\alpha_i M$.

To simplify the exposition, from now on we assume there are two firms doing research on the market. Their research intensities are denoted by $\alpha_1 \geq \alpha_2$. The degree of overlaps in information different firms acquire will have effect on the covariances of resulting signals. The following proposition summarizes these effects for the three types of research outlined above:

PROPOSITION 2.2 *Let two firms with research efforts* $\alpha_1 \geq \alpha_2$ *obtain signals* $\tilde{s}_1$ *and* $\tilde{s}_2$*, respectively. Let the internal correlation be* $\hat{\rho} \in [0, 1)$ *and* $\rho = 2\hat{\rho}/(1 + \hat{\rho})$*. Then the* $Cov(\tilde{s}_1, \tilde{s}_2)$ *will be the following, depending on the type of research performed:*

$$Cov(\tilde{s}_1, \tilde{s}_2) = \Sigma_0 \rho \alpha_1 \alpha_2 \quad \text{— non-overlapping} \tag{2.4}$$

$$Cov(\tilde{s}_1, \tilde{s}_2) = \Sigma_0 \alpha_2 (1 - (1 - \alpha_1)\rho) \quad \text{— perfectly-overlapping} \tag{2.5}$$

$$Cov(\tilde{s}_1, \tilde{s}_2) = \Sigma_0 \alpha_1 \alpha_2 \quad \text{— randomly-overlapping} \tag{2.6}$$

*See Appendix A.2 for proof.*

Up to now we were assuming finite $M \in \mathbb{N}$ and restricting values for $\alpha$ to satisfy $\alpha M \in \mathbb{N}$. In the following analysis we want to model continuous research efforts and ideally we would like the domain for $\alpha$ to be $\mathbb{R} \cap [0, 1]$. We are not able to do this exactly, however by talking a very large number for $M$ almost any rational number $\alpha \in \mathbb{Q}$ will satisfy the needed condition $\alpha M \in \mathbb{N}$ and all the above proofs will go through. By taking $M \to \infty$ the above formulas work for any $\alpha \in \mathbb{R} \cap [0, 1]$, however the formal proof of this is out of scope of this paper. Also we need $M \to \infty$ for the randomly-overlapping research to produce jointly-normal signals, required for further analysis.[7]

## 3. TRADING ENVIRONMENT

We will apply the aforementioned research technologies to a trading environment in the spirit of Kyle [1985b]. This trading model will be used to demonstrate the key results of this paper, however the general approach to modeling learning process can be used in conjunction with other economic models featuring uncertainty.

We consider one-period trading model with a single asset traded by two research firms, a mass of uninformed liquidity traders and a competitive market-maker. All agents are assumed to be risk-neutral. The asset has ex post common liquidation value to all market participants, denoted by $\tilde{v}$. Ex ante prior knowledge is that $\tilde{v}$ is normally distributed: $\tilde{v} \sim N(0, \Sigma_0)$. Prior to trading each research firm learns a private signal – a realization $s_i$ of a random variable $\tilde{s}_i$, which may be correlated with $\tilde{v}$ and the other firm's signal.[8] Upon observing the signals, each firm submits market order $\tilde{x}_i = X_i(\tilde{s}_i)$ to market-maker, where $X_i(\cdot)$ is a measurable function with respect to $\tilde{s}_i$. The random variables $\tilde{v}$, $\tilde{s}_1$, $\tilde{s}_2$ are assumed to be jointly normally distributed with zero mean. Liquidity traders' aggregate order flow $u$ is a realization of a random variable $\tilde{u}$, which is assumed to be normally distributed: $\tilde{u} \sim N(0, \sigma_u^2)$ and independent of all random variables mentioned above. The market-maker accommodates all excessive order flow $\tilde{X} + \tilde{u} = \sum_{i=1}^{2} \tilde{x}_i + \tilde{u}$ and sets the price $\tilde{p} = P(\tilde{X} + \tilde{u})$, where $P(\cdot)$ is a measurable function with respect to total order flow $\tilde{X} + \tilde{u}$. Then the

[7] The proof of Proposition 2.2 involved the hypergeometric distribution for the size of overlap in the randomly-overlapping case, thus the distribution of signals was a mixture of jointly-normal distributions (which fails to be jointly-normal itself for finite $M$). Once $M \to \infty$ the mixing does not matter and we obtain jointly-normal distribution of signals.

[8] We will use a particular research technology to determine the joint distribution of $\tilde{v}$ and firms' signals later on.

profits for each research firm are $\tilde{\pi}_i = (\tilde{v} - \tilde{p})\tilde{x}_i = \pi_i(X_1, X_2, P)$. Profit of one research firm depends on the other firm's trading strategy through the total order flow and its effect on price.

DEFINITION 3.1 *Similar to Kyle [1985b] a* trading equilibrium *is a set of functions* $X_1, X_2, P$ *that satisfy the two conditions:*
Profit Maximization: *for any research firm* $i \in \{1, 2\}$, *any alternative trading strategy* $X'$ *and any realization of* $i$*-th signal* $s_i$:[9]

$$E(\pi_i(X_i, X_{-i}, P)|\tilde{s}_i = s_i) \geq E(\pi_i(X', X_{-i}, P)|\tilde{s}_i = s_i) \tag{3.1}$$

Market Efficiency: $P$ *is such that random variable* $\tilde{p}$ *satisfies the following:*[10]

$$\tilde{p} = E(\tilde{v}|\tilde{X} + \tilde{u}) \tag{3.2}$$

In order to find the trading equilibrium defined above we need to parameterize the covariance matrix of the $\tilde{v}$, $\tilde{s}_1$, $\tilde{s}_2$ joint normal distribution. Throughout the paper we will apply the following notation:

$$\begin{aligned} \Lambda_i &= Var(\tilde{s}_i) \\ c_i &= Cov(\tilde{s}_i, \tilde{v}) \\ \Omega &= Cov(\tilde{s}_1, \tilde{s}_2) \end{aligned}$$

Given this parametrization we have the following:

$$\begin{pmatrix} \tilde{v} \\ \tilde{s}_1 \\ \tilde{s}_2 \end{pmatrix} \sim N\left(\mathbf{0}, \Psi = \begin{pmatrix} \Sigma_0 & c_1 & c_2 \\ c_1 & \Lambda_1 & \Omega \\ c_2 & \Omega & \Lambda_2 \end{pmatrix}\right) \tag{3.3}$$

In the spirit of Kyle [1985b] and other similar models, we will concentrate on the linear trading equilibrium, characterized by linear functions $X_1$, $X_2$

[9]Having only two research firms in the model, we will denote the "opponent" by {-i} index.

[10]The market efficiency condition is implied by the market-makers' competition that drives expected profit of the market-maker to zero.

and linear pricing rule $P$.[11] The following proposition characterizes such equilibrium for any permissible covariance matrix $\Psi$:

PROPOSITION 3.1 *Given any permissible $\Psi$ there exists a unique linear trading equilibrium defined by $X_i = (\beta_i/\lambda)\tilde{s}_i$ for each $i \in \{1,2\}$ and $P = \lambda(\tilde{X} + \tilde{u})$ where the constants $\beta_i$ and $\lambda$ are:*

$$\beta_i = \frac{2\Lambda_{-i}c_i - \Omega c_j}{4\Lambda_1\Lambda_2 - (\Omega)^2} \tag{3.4}$$

$$\lambda = \frac{1}{\sigma_u}\sqrt{\Lambda_1\beta_1^2 + \Lambda_2\beta_2^2} \tag{3.5}$$

*See Appendix B.1 for proof.*

### 3.1. *Endogenizing Research Efforts*

A research technology will determine the properties of firms' signals (covariance matrix $\Psi$ from (3.3)), and given these properties a unique linear trading equilibrium will arise according to Proposition 3.1. In this section we assume that both firms choose their research efforts $\alpha_i \in [0,1]$ simultaneously in the beginning of the trading game and then learn realizations of their signals $\tilde{s}_i$. We want to study incentives of each firm to stay in a given trading equilibrium, and for this purpose we need to know what happens to the trading profit when a firm secretly obtains a signal of a different quality. We assume that such a deviation is unobservable for all other market participants.

DEFINITION 3.2 *Denote by $\Psi^\alpha = \Psi(\alpha_1,\alpha_2)$ the covariance matrix of firms' signals $\tilde{s}_i(\alpha_i)$ and the true value $\tilde{v}$ (3.3). Denote by $(X_1^\alpha, X_2^\alpha, P^\alpha)$ the unique linear trading equilibrium for $\Psi^\alpha$. Research intensities $(\alpha_1,\alpha_2)$ constitute* a

[11] The existence of non-linear trading equilibria in Kyle-type framework is an open question.

Nash equilibrium of the game *if* $\forall\hat{\alpha} \in [0,1]$ *and* $\forall i \in \{1,2\}$ *the following condition holds:*

$$\max_{X_i}\left[E(\pi_i(X_i, X_{-i}^{\alpha}, P^{\alpha})|\tilde{s}_i(\hat{\alpha}) = s_i)\right] - c(\hat{\alpha}) \leq \tag{3.6}$$
$$\leq E(\pi_i(X_i^{\alpha}, X_{-i}^{\alpha}, P^{\alpha})|\tilde{s}_i(\alpha_i) = s_i) - c(\alpha_i)$$

The condition 3.6 highlights two important points: 1) when one firm chooses an off-equilibrium research effort $\hat{\alpha}$, all other market participants do not observe that move; 2) the deviating firm is able to reoptimize its trading strategy $X_i$ given the new properties of the obtained signal $\tilde{s}_i(\hat{\alpha})$. In the following lemma we derive the reoptimized trading strategy for the deviating firm and the associated expected profits. Then we provide the key existence and uniqueness results for the defined Nash equilibrium.

Lemma 3.1 *Suppose firm i enters the trading game with an arbitrary signal* $\tilde{s}_i^{\text{new}}$*, characterized by parameters* $(\tilde{\Lambda}_i, \tilde{c}_i, \tilde{\Omega})$*. The opponent and the market-maker do not observe the quality of the new signal, instead they follow a given trading equilibrium for some covariance matrix* $\Psi$ *(their belief about* $\tilde{s}_i$ *may be inconsistent with the true quality of* $\tilde{s}_i$*). Then firm i's optimal trading strategy is* $X_i^{\text{new}} = (\beta_i^{\text{new}}/\lambda)s_i^{\text{new}}$ *where:*

$$\beta_i^{\text{new}} = \tilde{c}_i/(2\tilde{\Lambda}_i) - \tilde{\Omega}/(2\tilde{\Lambda}_i)\beta_{-i} \tag{3.7}$$

*and expected profit is:*

$$E(\pi_i) = \frac{1}{4\lambda\tilde{\Lambda}_i}\left(\tilde{c}_i - \tilde{\Omega}\beta_{-i}\right)^2 \tag{3.8}$$

*where* $\beta_{-i}$ *and* $\lambda$ *are determined according to Proposition 3.1.*
*See Appendix B.2 for proof.*

In the following proposition we assume that cost function is linear and marginal costs of doing research are constant. One may repeat the analysis for a different choice of cost function, however we believe that a linear cost function is a reasonable assumption given the way we model research technologies. In section 2 we assumed that information coming from different sources is identically distributed, thus it is natural to assume equal extraction costs for identical pieces of information.

PROPOSITION 3.2 *Assume linear costs of doing research and define* $mc = c'(\alpha) \geq \underline{mc}$.[12] *Define the set of model parameters as* $\Theta = (\Sigma_0, \sigma_u^2, \hat{\rho}, mc)$. *Then for any* $\Theta$ *satisfying the restrictions outlined above, we have:*

1. *Under* ***non-overlapping*** *research there exists a unique Nash equilibrium* $\alpha_1 = \alpha_2 = \alpha^{no}$ *and it is symmetric.*[13]
2. *Under* ***randomly-overlapping*** *research there are two cases possible:*

   **when** $\hat{\rho} > 1/3$ *there exists a unique Nash equilibrium* $\alpha_1 = \alpha_2 = \alpha^{ro}$, *it is symmetric, and* $\alpha^{ro} < \alpha^{no}$

   **when** $\hat{\rho} < 1/3$ *there exist three Nash equilibria:*

   *(a) a symmetric Nash equlibrium* $\alpha_1 = \alpha_2 = \alpha^{ro}$, $\alpha^{ro} < \alpha^{no}$

   *(b) two asymmetric Nash equilibria of the form* $\alpha_i > \alpha^{no}, \alpha_{-i} = 0$ *(we will refer to these as* corner equilibria*)*

3. *Under* ***perfectly-overlapping*** *research there exist only two Nash equilibria of the form* $\alpha_i > \alpha_{-i}$ *and when* $\hat{\rho} \leq 1/3$ *both equilibria are corner equilibria with* $\min(\alpha_i, \alpha_{-i}) = 0$.

*See Appendix B.3 for proof.*

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[12]The lower-bound on $mc$ ensures that costs are high enough, so that in a single-firm version of the trading game it is not optimal for the firm to explore all sources of information, that is $\alpha^* < 1$.

[13]If $2\alpha^{no} > 1$ for some parameter values $\Theta$ then the non-overlapping research assumption is no longer valid.

## 4. NUMERICAL ANALYSIS AND DISCUSSION

In this section we demonstrate the arising equilibria for two different scenarios. According to Proposition 3.2 the existence of corner equilibrium in the randomly-overlapping case, as well as the nature of asymmetric equilibrium in perfectly-overlapping case — both depend on the magnitude of internal correlation of information $\hat{\rho}$. This correlation parameter can be thought of as a measure of similarity of information coming from different sources (see Definition 2.2). In the following tables we solve for arising Nash equilibria when the information is not very similar (Table I) and the opposite case when it is highly similar (II).

The general insight is that low internal correlation of information ($\hat{\rho} < 1/3$) together with overlaps in research process lead to emergence of corner Nash equilibrium that is highly preferred over the symmetric equilibrium by the firm remaining in business. This suggests that if we introduce timing in making unobservable research decisions, there will be a huge first-mover advantage and associated barriers to entry for the competitor. When research is non-overlapping only symmetric equilibrium exists, both research firms earn positive profits and thus new firms can easily enter the industry as in the model by Dierker [2006]. However, when the internal correlation of information is low and there are overlaps in firms' research, the multiplicity of equilibria arises (consider randomly-overlapping case in Table I). The corner equilibrium would be preferred over the symmetric one by the firm remaining in business (in our numerical example profits of the first firm increase more than twice). The increase in profits for the monopoly research firm is an anticipated result — it is well-known that in a Kyle-type trading competition between firms with correlated signals leads to more aggressive trading and lower profits. Having research efforts endogenous we see that in a corner equilibrium the monopolist acquires almost twice more information (0.7 versus 0.37). Also for the same marginal cost of research more sources of information are explored in corner equilibrium comparing to symmetric equilibrium under randomly-overlapping research (0.7 versus $2 * 0.37 - 0.37^2 = 0.6$). And under perfectly-overlapping research the corner equilibrium is the only situation possible. It can be seen that the degree of overlaps in research process can explain the existence of such a corner equilibrium and associated barriers to entry.

When we increase the internal correlation above the 1/3 benchmark we obtain the results in Table II. It can be seen that with high similarity of information coming from different sources there is no way one research firm can drive the other out in the equilibrium. All firms are able to get higher profits from trading with lower research effort, because the first bits of information they acquire are highly informative about the remaining unexplored sources of information.

In our analysis we used a two-firm trading model. Nevertheless, the above logic can be extended to a multi-firm setting. In a two-firm setting informational barriers to entry are high enough to drive the second research firm out of the business when $\hat{\rho} < 1/3$. With more research firms operating on the market, the possibility and the extent of overlaps is greater (there are more firms to overlap with), thus even for higher values of internal correlation $\hat{\rho} > 1/3$, the oligopolistic structure of research industry may still be sustained. While two firms can coexist in a symmetric equilibrium for $\hat{\rho} > 1/3$, the third possible entrant may be driven out for the same reasons discussed above.

Only non-overlapping research technology can sustain a symmetric equilibrium with large number of research firms. When all these firms are similar to hedge-funds having unique information to trade on, as long as these hedge-funds so not overlap in doing research there may be quite a few of them in equilibrium. But in this case the variety of sources of information must also be high enough, so that the necessary restriction on non-

TABLE I

LOW INTERNAL CORRELATION $\hat{\rho} = 0.25$ ($< 1/3$ BENCHMARK)

| | Corner Equilibrium | | Symmetric Equilibrium | |
|---|---|---|---|---|
| Type of Research | $\alpha_i$ | $E(\pi_i)$ | $\alpha_i$ | $E(\pi_i)$ |
| Non- | – | – | 0.42 | 100% |
| Overlapping | – | – | 0.42 | 100% |
| Randomly | 0.7 | 272.17% | 0.37 | 75.25% |
| Overlapping | 0 | 0% | 0.37 | 75.25% |
| Perfectly | 0.7 | 272.17% | – | – |
| Overlapping | 0 | 0% | – | – |

The parameter values used: $\Sigma_0 = 1$, $\sigma_u^2 = 1$, $\rho = 0.4$ (corresponds to $\hat{\rho} = 0.25$). The profit of each firm in the symmetric equilibrium for non-overlapping research is 0.0521 and is normalized to 100% in the table. The marginal cost of research is 0.4344 and set so that in the corner equilibrium the optimal research effort is 0.7.

overlapping research technology holds ($\sum_i \alpha_i < 1$). When it is not the case, overlaps in information appear and the barriers to entry discussed above kick in.

TABLE II

HIGH INTERNAL CORRELATION $\hat{\rho} = 0.6$ ($> 1/3$ BENCHMARK)

| | Asymmetric Equilibrium | | Symmetric Equilibrium | |
|---|---|---|---|---|
| Type of Research | $\alpha_i$ | $E(\pi_i)$ | $\alpha_i$ | $E(\pi_i)$ |
| Non- | – | – | 0.28 | 197.16% |
| Overlapping | – | – | 0.28 | 197.16% |
| Randomly | – | – | 0.268 | 179.78% |
| Overlapping | – | – | 0.268 | 179.78% |
| Perfectly | 0.445 | 315.21% | – | – |
| Overlapping | 0.104 | 26.97% | – | – |

The parameter values used: $\Sigma_0 = 1$, $\sigma_u^2 = 1$, $\rho = 0.75$ (corresponds to $\hat{\rho} = 0.6$). For the sake of comparability with Table I, the same value of 0.0521 for profits is normalized to 100%. The marginal cost of research is 0.4344 (same as used in Table I).

## 5. CONCLUSION

In this paper we provided a novel approach to modeling research. The approach allowed to account explicitly for the correlation of information coming from various sources and the possibility of overlaps in learning. As a possible application of the approach, we studied the Kyle-type insider trading model and showed that overlaps in doing market research indeed had significant entry-deterrence effect and could explain the oligopolistic structure of the industry, as well as cross-sectional variation in analyst coverage.

# APPENDIX A: RESEARCH TECHNOLOGIES

## A.1. *Proposition 2.1*

PROOF: Equation (2.1) together with the $Var(\tilde{v}) = \Sigma_0$ implies:

$$MVar(\tilde{e}_i) + M(M-1)Cov(\tilde{e}_i, \tilde{e}_j) = \Sigma_0$$
$$Var(\tilde{e}_i) = (1/M)(\Sigma_0 - M(M-1)Cov(\tilde{e}_i, \tilde{e}_j)) \tag{A.1}$$

Now use definition of a signal (2.2) together with the above equation (A.1) to calculate $Var(\tilde{s}_i)$ and $Cov(\tilde{s}_i, \tilde{v})$:

$$Var(\tilde{s}_i) = \alpha\Sigma_0 - \alpha(1-\alpha)M^2 Cov(\tilde{e}_i, \tilde{e}_j) \tag{A.2}$$
$$Cov(\tilde{s}_i, \tilde{v}) = Var(\tilde{s}_i) + \alpha M(M - \alpha M)Cov(\tilde{e}_i, \tilde{e}_j) = \alpha\Sigma_0 \tag{A.3}$$

Equation (A.3) is the result we need. In order to simplify equation (A.2) we note that $Var(\tilde{s}_i)$ must not depend on $M$ according to the consistency argument discussed in Section 2. Thus as $M$ increases, covariance of information bits $\tilde{e}_i$ and $\tilde{e}_j$ must decay at the rate $M^2$, in other words $Cov(\tilde{e}_i, \tilde{e}_j) = const/M^2$. The internal correlation parameter $\rho$ can be used to pin down the unknown value of *const* (in the following we use definition of internal correlation, Definition 2.2):

$$\hat{\rho} = (const/4)/(0.5\Sigma_0 - const/4)$$
$$const = \Sigma_0(2\tilde{\rho}/(1+\hat{\rho})) = \Sigma_0\rho$$

Thus the internal correlation parameter together with consistency argument determine the covariance of bits of information $Cov(\tilde{e}_i, \tilde{e}_j) = \Sigma_0\rho/M^2$. Plugging this expression in equation A.2 obtains the result. *Q.E.D.*

## A.2. *Proposition 2.2*

PROOF: We will use the results from A.1, in particular the result for $Cov(\tilde{e}_i, \tilde{e}_j) = \Sigma_0\rho/M^2$. There are two firms doing research, $\alpha_1 \geq \alpha_2$, and assume both $\alpha_1 M$ and $\alpha_2 M$ are Natural numbers. For non-overlapping research there are no common bits of information included in both firms' signals, thus:

$$Cov(\tilde{s}_1, \tilde{s}_2) = (\alpha_1 M)(\alpha_2 M)Cov(\tilde{e}_i, \tilde{e}_j) = \Sigma_0 \rho \alpha_1 \alpha_2 \tag{A.4}$$

For perfectly-overlapping research we can rewrite the first firm's signal as:

$$\tilde{s}_1 = \tilde{s}_2 + \sum_{j=1}^{(\alpha_1-\alpha_2)M} \tilde{e}_j$$

And thus we have:

$$Cov(\tilde{s}_1, \tilde{s}_2) = Var(\tilde{s}_2) + (\alpha_2 M)(\alpha_1 M - \alpha_2 M)Cov(\tilde{e}_i, \tilde{e}_j)$$

Using the first equation in (2.1) to substitute for $Var(s_2)$ we obtain the result:

$$Cov(\tilde{s}_1, \tilde{s}_2) = \Sigma_0 \alpha_2 (1 - (1 - \alpha_1)\rho) \tag{A.5}$$

To derive the needed expression for randomly-overlapping research assume $aM$ bits of information constitute the overlap of the two firms' signals, $a \leq \alpha_2$. Similarly to perfectly-overlapping case, we can rewrite firms' signals as the sum of overlapping and non-overlapping parts, where $aM$ is the size of overlapping part. We denote the overlapping part of both signals by $s_3$. Holding $a$ fixed we can express $Cov(\tilde{s}_i, \tilde{s}_j)$ in terms of $a$:

$$Cov(\tilde{s}_1, \tilde{s}_2|a) = Var(\tilde{s}_3) + (a(\alpha_1 - a) + a(\alpha_2 - a) + (\alpha_1 - a)(\alpha_2 - a))M^2 Cov(\tilde{e}_i, \tilde{e}_j)$$

The above expression simplifies to:

$$Cov(\tilde{s}_1, \tilde{s}_2|a) = a\Sigma_0(1 - \rho) + \alpha_1 \alpha_2 \Sigma_0 \rho$$

When each firms makes independent random draws of size $\alpha_i M$, the expected size of overlap is $E(a) = \alpha_1 \alpha_2$. To get this result, imagine firm 1 moves first and selects $\alpha_1 M$ sources of information, assume all numbers are Natural in the following discussion. Then firm 2 draws randomly $\alpha_2 M$ sources without replacement, thus the number of sources drawn by both firms will follow hypergeometric distribution with parameters $N = N, m = \alpha_1 N, n = \alpha_2 M$. The expected value of hypergeometric distribution is $mn/N$, which is exactly $\alpha_1\alpha_2 N$ in our case. Thus $E(a) = \alpha_1\alpha_2$ and the unconditional covariance of signals is:

$$Cov(\tilde{s}_1, \tilde{s}_2) = E(a)\Sigma_0(1-\rho) + \alpha_1\alpha_2\Sigma_0\rho = \alpha_1\alpha_2\Sigma_0 \tag{A.6}$$

*Q.E.D.*

## APPENDIX B: TRADING ENVIRONMENT

### B.1. *Proposition 3.1*

PROOF: Conjecture linear trading strategies $X_i = \tilde{\beta}_i\tilde{s}_i$ for the two firms and linear pricing rule of the form $P = \lambda(\tilde{X} + \tilde{u})$. Start with the profit maximization condition (3.1):

$$\max_{X_i}\{E(\pi_i(X_i, X_{-i}, P)|\tilde{s}_i = s_i)\} = \max_x\left\{E((v - \lambda(x + \tilde{\beta}_{-i}s_{-i} + \tilde{u}))x|\tilde{s}_i = s_i)\right\}$$

Following the approach of Bernhardt and Taub [2008] we rewrite the $i$-th firm problem as unconditional maximization with respect to its trading intensity $\tilde{\beta}_i$:

$$\max_x\left\{E((v - \lambda(x + \tilde{\beta}_{-i}s_{-i} + \tilde{u}))x|\tilde{s}_i = s_i)\right\} = \max_\beta\left\{E((v - \lambda(\beta s_i + \tilde{\beta}_{-i}s_{-i} + \tilde{u}))\beta s_i)\right\}$$

Using the joint-normality assumption (3.3) and independence of liquidity trade the unconditional problem simplifies to:

$$\max_\beta\left\{\beta c_i - \lambda\beta^2\Lambda_i - \lambda\beta\tilde{\beta}_{-i}\Omega\right\} \tag{B.1}$$

Assuming $\lambda$ is positive, the second-order condition for the above maximization is satisfied. Thus, it is sufficient to consider the system of two first-order conditions for two firms:

$$\begin{pmatrix} 1 & \Omega/(2\Lambda_1) \\ \Omega/(2\Lambda_2) & 1 \end{pmatrix} \begin{pmatrix} \tilde{\beta}_1 \\ \tilde{\beta}_2 \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} c_1/(2\Lambda_1) \\ c_2/(2\Lambda_2) \end{pmatrix}$$

According to this system $\lambda\tilde{\beta}_i$ depends on model parameters only and does not depend on conjectured value of $\lambda$. Thus it is convenient to define a new constant $\beta_i = \lambda\tilde{\beta}_i$ and work with a pair $(\beta_i, \lambda)$ from now on:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1 & \Omega/(2\Lambda_1) \\ \Omega/(2\Lambda_2) & 1 \end{pmatrix}^{-1} \begin{pmatrix} c_1/(2\Lambda_1) \\ c_2/(2\Lambda_2) \end{pmatrix}$$

From the above we obtain the expression (3.4) for $\beta_i$ presented in the proposition. Note that with conjectured linear trading and pricing rules the $\beta_i$ constants are determined uniquely.

Now use the market efficiency condition (3.2) to determine $\lambda$. Note that if $\lambda$ is determined uniquely, the initial trading intensities $\tilde{\beta}_i = \lambda\beta_i$ will be unique. Under the conjectured linear trading strategies the total order flow is:

$$\tilde{X} + \tilde{u} = (\beta_1/\lambda)\tilde{s}_1 + (\beta_2/\lambda)\tilde{s}_2 + \tilde{u}$$

and is jointly normally distributed with $\tilde{v}$. This implies linearity of pricing rule and we can treat the market efficiency condition (3.2) as a linear regression of $\tilde{v}$ on $\tilde{X} + \tilde{u}$, where $\lambda$ must satisfy:

$$\lambda = \frac{Cov(\tilde{v}, \tilde{X} + \tilde{u})}{Var(\tilde{X} + \tilde{u})} = \frac{(\beta_1/\lambda)c_1 + (\beta_2/\lambda)c_2}{(\beta_1/\lambda)^2\Lambda_1 + (\beta_2/\lambda)^2\Lambda_2 + 2(\beta_1/\lambda)(\beta_2/\lambda)\Omega + \sigma_u^2}$$

$$(\lambda\sigma_u)^2 = \beta_1(c_1 - \beta_1\Lambda_1 - \beta_2\Omega) + \beta_2(c_2 - \beta_2\Lambda_2 - \beta_1\Omega)$$

Using expression (3.4) it can be shown that (alternatively, it is implied by the first order condition for the firm's problem B.1):

$$c_i - \beta_i\Lambda_i - \beta_{-i}\Omega = \Lambda_i\beta_i$$

Plugging this result in the above equation we obtain expression (3.5) for $\lambda$. The linear trading equilibrium is unique. *Q.E.D.*

## B.2. *Lemma 3.1*

PROOF: Firm $i$'s opponent and the market-maker will follow the given trading equilibrium, and their behavior will be captured by constants $\beta_{-i}$ and $\lambda$ according to Proposition 3.1. Firm $i$'s profit maximization problem is:

$$\max_x \left\{ E((v - \lambda(x + \tilde{\beta}_{-i} s_{-i} + \tilde{u}))x | \tilde{s}_i = s_i) \right\}$$

Following the approach in Bernhardt and Taub [2008] and in the proof of Proposition 3.1, the unconditional maximization problem simplifies to:

$$\max_\beta \left\{ \beta \tilde{c}_i - \lambda \beta^2 \tilde{\Lambda}_i - \beta \beta_{-i} \tilde{\Omega} \right\}$$

The first-order condition is sufficient and gives the expression (3.7) for $\beta_i^{\text{new}}$:

$$\beta_i^{\text{new}} = \tilde{c}_i/(2\tilde{\Lambda}_i) - \tilde{\Omega}/(2\tilde{\Lambda}_i)\beta_{-i}$$

Plugging the solution for $\beta$ into the unconditional maximization problem we obtain the expression (3.8) for expected profit:

$$\begin{aligned} E(\pi_i) &= \frac{\tilde{c}_i^2}{2\lambda\tilde{\Lambda}_i} - \frac{\tilde{\Omega}\tilde{c}_i}{2\tilde{\Lambda}_i}\beta_{-i} - \lambda\tilde{\Lambda}_i \left( \frac{\tilde{c}_i^2}{2\lambda^2\tilde{\Lambda}_i^2} + \frac{\tilde{\Omega}^2}{4\lambda^2\tilde{\Lambda}_i^2}\beta_{-i}^2 - \frac{\tilde{c}_i\tilde{\Omega}}{2\lambda^2\tilde{\Lambda}_i^2}\beta_{-i} \right) \\ &- \lambda\tilde{\Omega} \left( \frac{\tilde{c}_i}{2\lambda^2\tilde{\Lambda}_i}\beta_{-i} - \frac{\tilde{\Omega}}{2\lambda^2\tilde{\Lambda}_i}\beta_{-i}^2 \right) = \\ &= \frac{1}{4\lambda\tilde{\Lambda}_i} \left( \tilde{\Omega}^2\beta_{-i}^2 - 2\tilde{\Omega}\tilde{c}_i\beta_{-i} + \tilde{c}_i^2 \right) = \\ &= \frac{1}{4\lambda\tilde{\Lambda}_i} \left( \tilde{c}_i - \tilde{\Omega}\beta_{-i} \right)^2 \end{aligned}$$

*Q.E.D.*

## B.3. *Proposition 3.2*

PROOF: Firstly,[14] consider equilibria characterized by interior choices for both firms' research efforts $\alpha_1 > 0, \alpha_2 > 0$. Use equation 3.8 in Lemma 3.1

[14] This is the general outline of the proof. Detailed derivations of all results discussed below are available from the author upon request.

to rewrite firm's expected payoff function $E(\ddot{\pi})$ in terms of its research effort $\alpha_i$ (holding $\alpha_{-i}$, $\lambda$ and $\beta_{-i}$ constant):

$$E(\ddot{\pi}_i(\alpha_i)) = \frac{1}{4\lambda\tilde{\Lambda}_i(\alpha_i)}\left(\tilde{c}_i(\alpha_i) - \tilde{\Omega}(\alpha_i)\beta_{-i}\right)^2 - (mc)\alpha_i \tag{B.2}$$

Plug the functional forms for $\tilde{\Lambda}_i(\alpha_i)$, $\tilde{c}_i(\alpha_i)$ and $\tilde{\Omega}_i(\alpha_i)$ under the non-overlapping research assumption (use Propositions 2.1 and 2.2). The first order condition for payoff maximization (equation B.2) simplifies to:

$$MB_i(\alpha_i) = \frac{\Sigma_0(1-\rho)}{4\lambda}\left(\frac{1-\rho\alpha_{-i}\beta_{-i}}{1-(1-\alpha_i)\rho}\right)^2 = mc \tag{B.3}$$

Similarly, we obtain the first order conditions for randomly-overlapping (B.4) and perfectly-overlapping (B.5) research technologies:

$$MB_i(\alpha_i) = \frac{\Sigma_0(1-\rho)}{4\lambda}\left(\frac{1-\alpha_{-i}\beta_{-i}}{1-(1-\alpha_i)\rho}\right)^2 = mc \tag{B.4}$$

$$\alpha_1 \geq \alpha_2 \tag{B.5}$$

$$MB_1(\alpha_1) = \frac{\Sigma_0(1-\rho)}{4\lambda}\left(\frac{1}{1-(1-\alpha_1)\rho} - \frac{\alpha_2}{\alpha_1}\beta_2\right)\left(\frac{1}{1-(1-\alpha_1)\rho} + \frac{\alpha_2}{\alpha_1}\beta_2\right) = mc$$

$$MB_2(\alpha_2) = \frac{\Sigma_0(1-\rho)}{4\lambda}\left(\frac{1-\beta_1+\beta_1(1-\alpha_1)\rho}{1-(1-\alpha_2)\rho}\right)^2 = mc$$

It can be seen that for all three cases $MB_i$ is a decreasing function of $\alpha_i$, thus the maximization problem is concave and the first order condition is sufficient to characterize the interior solution for optimal research effort.[15] Any candidate Nash equilibrium with positive $\alpha_1$ and $\alpha_2$ must satisfy the above first order conditions for both firms.

Equate the candidate equilibrium values of $MB_1$ and $MB_2$ (due to the constant $mc$ assumption) and determine the relationship between $\alpha_1$ and $\alpha_2$

[15] A slightly different argument applies to the first condition of the perfectly-overlapping case: it can be shown that $MB_1 > MB_2$, while $MB_1$ has at most one point where it changes direction. Thus although the first firm's maximization problem is not concave, the first order condition is still sufficient.

for each of the three types of research. The non-overlapping case simplifies to:

$$2\alpha_1 + \alpha_2 = 2\alpha_2 + \alpha_1$$

This means that if we look for an equilibrium with both firms having strictly positive research intensities $\alpha > 0$, then it must be the case that $\alpha_1 = \alpha_2$ and the equilibrium is symmetric.[16] The randomly-overlapping case yields a similar expression:[17]

$$2\rho\alpha_1 + \alpha_2 = 2\rho\alpha_2 + \alpha_1$$

The perfectly-overlapping case simplifies to the following relationship between $\alpha_1$ and $\alpha_2$.

$$\{-2\rho(1-\rho(1+\alpha_1))\}\alpha_2^2 + \{2(1-\rho)(3\rho\alpha_1 - (1-\rho))\}\alpha_2 + \\ +\{\alpha_1(4(1-\rho)^2 - (1-\rho(1-\alpha_1))^2)\} = 0$$

We can solve the above equation for $\alpha_2$. It turns out that when $\rho \in (0.5, 1)$ and $\alpha_1\rho > 1-\rho$ we have positive $\alpha_2 < \alpha_1$. When one of the conditions is not satisfied, we have $\forall\alpha_2 \in [0,1]$ the first firm with higher $\alpha_1 > \alpha_2$ has $MB_1 > MB_2$, thus only corner solution is possible for $\alpha_2$ and there exist only two corner equilibria outlined in proposition 3.2.

(B.6) when $\rho \in (0.5, 1)$ and $\alpha_1\rho > 1-\rho$

$$\alpha_2 = \frac{-(1-\rho)(3\alpha_1\rho - 1 + \rho) + (\alpha_1\rho + 1 - \rho)\sqrt{(\alpha_1\rho - 1 + \rho)^2 + \rho^2\alpha_1^2}}{2\rho(\alpha_1\rho - 1 + \rho)}$$

(B.7) when $\rho \in (0, 0.5]$ or $\alpha_1\rho \le 1-\rho$

$$\alpha_2 = 0$$

[16] Most literature on multi-agent Kyle models considered exclusively the symmetric equilibria. This result may be interpreted as a justification for that.

[17] There is an indeterminacy arising for the knife-edge parameter value $\rho = 0.5$. We have not studied that case yet, so in this version of the paper we assume away this possibility.

Now we establish the existence and uniqueness of symmetric equilibria for non-overlapping and perfectly-overlapping cases. We plug the trading equilibrium values for $\beta_i$ and $\lambda$ into the first order conditions and simplify. Under non-overlapping research an interior symmetric equilibrium must satisfy the following:

$$MB^* = \frac{\sqrt{\Sigma_0 \sigma_u^2}(1-\rho)}{\sqrt{2}\sqrt{\alpha(1-(1-\alpha)\rho)}(3\alpha\rho + 2(1-\rho))} = mc$$

It can be seen that $MB^*$ is a decreasing function of $\alpha$ and as $\alpha \to 0$ we have $MB^* \to +\infty$, thus the symmetric equilibrium with $\alpha \in (0,1)$ exists for any value of $mc > \underline{mc}$. It can be shown that the imposed condition on $mc$ rules out the possibility of a fully-informed symmetric equilibrium with $\alpha = 1$ under non-overlapping research.

Similarly for symmetric equilibrium under randomly-overlapping research the necessary condition is:

$$MB^* = \frac{\sqrt{\Sigma_0 \sigma_u^2}(1-\rho)}{\sqrt{2}\sqrt{\alpha(1-(1-\alpha)\rho)}(\alpha(2\rho+1) + 2(1-\rho))} = mc$$

Similar argument establishes the existence and uniqueness of a symmetric equilibrium with $\alpha \in (0,1)$ for any value of $mc > \underline{mc}$.

For the perfectly-overlapping case we need to consider $\rho > 0.5$ and $\rho \leq 0.5$ separately. When $\rho > 0.5$ we have established that $\alpha_2 > 0$ in a candidate equilibrium, and $\alpha_2$ is uniquely determined by $\alpha_1$ according to equation B.6. It is difficult to demonstrate the existence and uniqueness result algebraically for the asymmetric equilibrium, however a numerical proof is available from the author upon request. When $\rho \leq 0.5$ we have the corner equilibrium and there is essentially only one research firm acting in the model. The condition establishing existence and uniqueness of the corner equilibrium is (in terms of positive research effort $\alpha_1 > 0$):

$$MB^* = \frac{\sqrt{\Sigma_0 \sigma_u^2}(1-\rho)}{2\sqrt{\alpha_1(1-(1-\alpha_1)\rho)}(\alpha_1\rho + (1-\rho))} = mc$$

Up to this point we have considered interior symmetric equilibria for non-overlapping and randomly-overlapping cases. To complete the proof we analyze the possibility of a corner equilibria arising for these two types of research. Firstly it can be shown that when a firm has an uninformative signal as a result of research effort $\alpha_2 = 0$, it is optimal not to participate in trading, $\beta_2 = 0$. The other firm's optimal trading strategy involves $\beta_1 = c_1/(2\Lambda_1)$. The corner equilibrium will satisfy the necessary condition 3.6 if $MB_2 \leq mc$. Under non-overlapping research technology this inequality is equivalent to:

$$\frac{\rho\alpha_1}{2(1-\rho)} \leq 0$$

It has no solution for $\alpha_1 > 0$ and $\rho \in (0,1)$ thus a corner equilibrium is not possible under non-overlapping research for all parameter values we consider. The corresponding inequality for randomly-overlapping research is:

$$\frac{(2\rho-1)\alpha_1}{2(1-\rho)} \leq 0$$

Thus there exists a corner equilibrium for $\rho < 0.5$, which corresponds to $\hat{\rho} < 1/3$. The existence and uniqueness argument is the same as for perfectly-overlapping research case for $\rho \leq 0.5$ (because in the corner equilibrium the only firm on the market has the same $MB^*$ irrespective of the type of research we consider).

This concludes the proof of the proposition.

*Q.E.D.*

REFERENCES

D. Bernhardt and B. Taub. Cross-asset speculation in stock markets. *The Journal of Finance*, 63(5):pp. 2385–2427, 2008. ISSN 00221082.

M. J. Brennan and A. Subrahmanyam. Investment analysis and price formation in securities markets. *Journal of Financial Economics*, 38(3):361 – 381, 1995. ISSN 0304-405X. .

M. Dierker. Endogenous information acquisition with cournot competition. *Annals of Finance*, 2(4):369–395, Oct. 2006.

C. W. Holden and A. Subrahmanyam. Long-lived private information and imperfect competition. *The Journal of Finance*, 47(1):pp. 247–270, 1992. ISSN 00221082.

A. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1335, Nov. 1985a.

A. S. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):pp. 1315–1335, 1985b. ISSN 00129682.

R. E. Verrecchia. Information acquisition in a noisy rational expectations economy. *Econometrica*, 50(6):pp. 1415–1430, 1982. ISSN 00129682.