95-771 Data Structures and Algorithms
Spring 2015

Midterm Exam

Full Name key

Coding Lists and Trees (18 points)

1. The following program contains four println() statements. Describe the output of the program when each of the println() statements is executed. (8 points):

```
package dsamidtermproject;

class Node {
  private int data;
  private Node next;
  public Node(int data, Node next) {
     this.data = data;
     this.next = next;
  }
  public int getData() {
     return data;
  }
  public void setData(int data) {
     this.data = data;
  }
  public Node getNext() {
     return next;
  }
  public void setNext(Node next) {
     this.next = next;
  }
}
public class DSAMidtermProject {
  public Node list = null;
  public void insert1(int x) {
     list = new Node(x,list);
  }
  public void insert2(int x) {
     if(list == null) insert1(x);
     else {
        Node m = list;
        Node p = list;
        while (m != null) {
           p = m;
           m = m.getNext();
        }
        p.setNext(new Node(x,null));
     }
  }
  public String toString() {
     String v = "";
     Node c = list;
     while (c != null) {
        v = v + c.getData();
        if(c.getNext()!= null) v = v + "->";
        c = c.getNext();
     }
     return v;
  }
```

add Front

add at end

33

18
no
12
16
18
15
21

100

Key

```
int xyz(Node p) {
   if ( p == null) return 0;
   else return xyz(p.getNext()) + 1;
}
int abc() {
   return xyz(list);
}

public static void main(String args[]) {
   DSAMidtermProject dsa = new DSAMidtermProject();
   for(int i = 0; i <= 5; i++) dsa.insert2(i);

   // Show output as 1.a
   System.out.println(dsa);

   // Show output as 1.b
   System.out.println(dsa.abc());

   dsa = new DSAMidtermProject();
   for(int i = 0; i <= 5; i++) dsa.insert1(i);

   // Show output as 1.c
   System.out.println(dsa);

   // Show output as 1.d
   System.out.println(dsa.abc());
  }
}
```

count

1.a What will the program display at the println marked 1.a? 0, 1, 2, 3, 4, 5

1.b What will the program display at the println marked 1.b? 6

1.c What will the program display at the println marked 1.c? 5, 4, 3, 2, 1, 0

1.d What will the program display at the println marked 1.d? 6

2 pts each

-0

2. The following program contains a System.out.print() statement in the traversal method. (10 points):

Key

```
package simpletree;
class Node {
   public int data;
   public Node lc;
   public Node rc;
   public Node(Node lc, int x, Node rc) {
      this.lc = lc;
      this.data = x;
      this.rc = rc;
   }
}
public class SimpleTree {
   public Node root;
   public static int ctr = 1;
   public SimpleTree() {
      root = null;
   }
   private Node add(Node t){
      if (t == null) {
         ctr = ctr * 2 ;
         return new Node(null,ctr,null);
      }
      t.lc = add(t.lc);
      t.rc = add(t.rc);
      return t;
   }
   public void add() {
      if (root == null) {
         ctr = ctr + 1;
         root = new Node(null,ctr,null);
      }
      else {
         add(root);
      }
   }
   public void traversal(Node t) {
      if(t != null) {
         traversal(t.lc);
         traversal(t.rc);
         System.out.print(t.data + " ");
      }
   }
   public void traversal() {
      traversal(root);
   }
```

ctr
1 ~~2~~ 4

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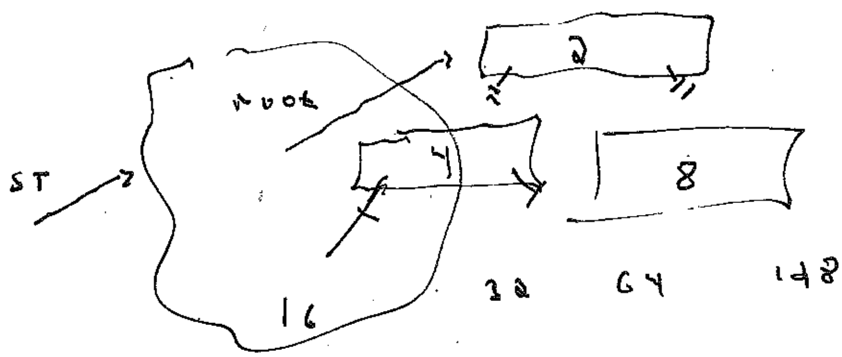
```
public static void main(String[] args) {

    SimpleTree st = new SimpleTree();
    st.add();
    st.add();

    // Question 2.a
    st.traversal();
    System.out.println();

    st.add();

    // Question 2.b
    st.traversal();
    System.out.println();

  }
}
```



2.a What will the program display at the traversal marked Question 2.a?

4, 8, 2

5 pts

2.b What will the program display at the traversal marked Question 2.b?

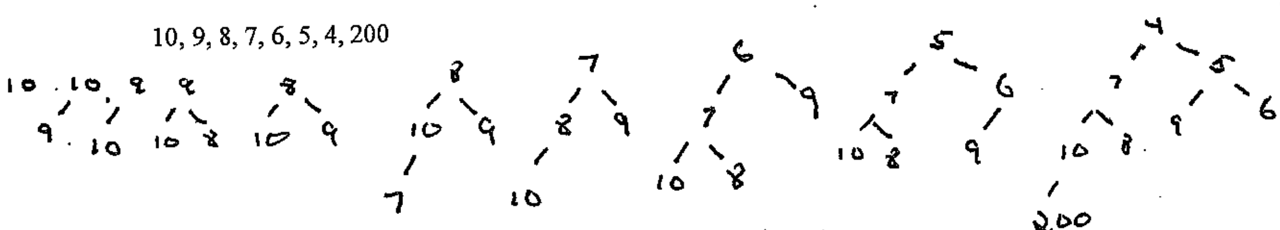
16, 32, 4, 64 128 8 2

5 pts

Heaps (12 points)

3) Insert the following 8 numbers into a min heap. Draw a new tree for each heap insertion. (4 Points)

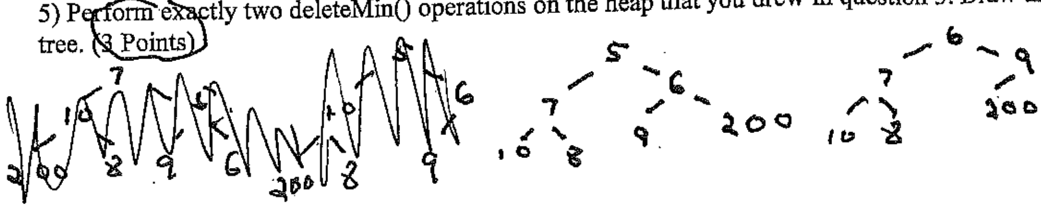
10, 9, 8, 7, 6, 5, 4, 200



Handle SNOW BALL

4) What is the height of the tree that you drew in question 3? (2 Points) 3

5) Perform exactly two deleteMin() operations on the heap that you drew in question 3. Draw the resulting tree. (3 Points)



6) Consider the following max heap implemented in an array. It is not quite correct. To make it a proper max heap exactly one swap must occur. What two numbers need to be swapped in order to make this a max heap? (3 points)

100
90
70
70
50
80
40
30
50
40

```
            100
          /     \
        90       70
       /  \     /  \
     70    50  80   40
    /  \   /
  30   50 40
```

Binary Trees (16 points)

7. Parts (a), (b), and (c) refer to the following binary tree:

```
        20
      /    \
    40      97
   /  \       \
  6    9       76
 / \    \     /  \
5   1    3   65   7
```

(a) List the data that would be accessed by a pre-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (3 points)

20, 40, 6, 5, 1, 9, 3, 97, 76, 65, 7

(b) List the data that would be accessed by an in-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (2 points)

5, 6, 1, 40, 9, 3, 20, 97, 65, 76, 7

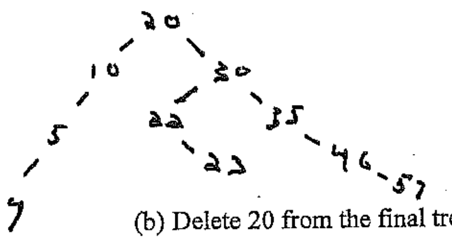
(c) List the data that would be accessed by a level-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (2 points)

20, 40, 97, 6, 9, 76, 5, 1, 3, 65, 7

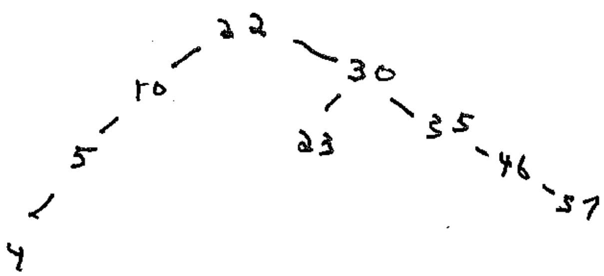
(d) In general, if a binary tree is perfectly balanced (unlike the tree pictured here) and complete with height h, how many leaves, in terms of h, will the tree have? (1 point) 2^h Note, this tree has a perfectly flat bottom. In addition, how many internal nodes would such a tree have (in terms of h)? (1 Point) $2^h - 1$

(e) In general, if a binary tree is perfectly balanced (unlike the tree pictured here) and complete with exactly k leaves. What is the height (in terms of k) of this tree? (2 points) $\text{Log}_2 K$
Note, this tree has a perfectly flat bottom.

8. (a) Insert he following numbers into a Binary Search Tree. Draw the tree after all insertions are complete. You need not show each step. (2 Points)
20, 10, 5, 4, 30, 35, 46, 57, 22, 23



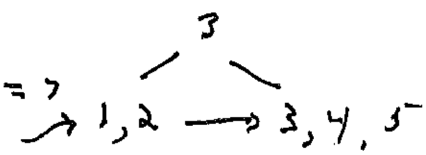
(b) Delete 20 from the final tree that you drew in 8 (a). Draw this final tree. (1 Points)



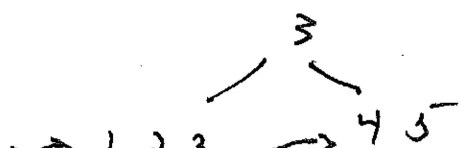
(c.) Insert the following numbers into a B+ tree. The min=2 and max = 4. Please note, this is a "B Plus" tree. (2 Points)

1,2,3,4,5

1, 2, 3, 4, 5 =>



or



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Project Questions (18 points)

9. Recall the Merkle-Hellman cryptosystem that we worked with in Project 1, the spell checker application in Project 2, and the graph coloring problem from Project 3.

Project 1 was based on the subset sum problem which is known to be NP-Complete. The problem itself can be described as follows: given a set of numbers *X* and a number **k**, is there a subset of *X*, which sums to k?

(a) Suppose X = {3,9,12,4,2} and **k** = 18. Is there a subset of X which sums to k?
Yes ____ Yes/No (2 points)

(b) The type of problem you were asked to solve in question 9 (a) is (Circle one answer): (2 Points)

1. an optimization problem.
2. a problem that is impossible to solve.
3. a problem that has been proven to take exponential time to solve.
4. a problem that has been proven to take factorial time to solve.
5. a decision problem.

(c) Suppose Alice sends a message (K) to Bob. K is computed using Bob's Merkle-Hellman public key combined with the message M. The central idea behind Merkle-Hellman is that a potential eaves dropper could read the message M if the eaves dropper could (2 Points)

1. Find K so that M is prime.
2. Modify Bob's public key.
3. Modify the super increasing sequence.
4. Find a subset of a super increasing sequence that sums to K.
5. Find a subset of Bob's public key that sums to K.

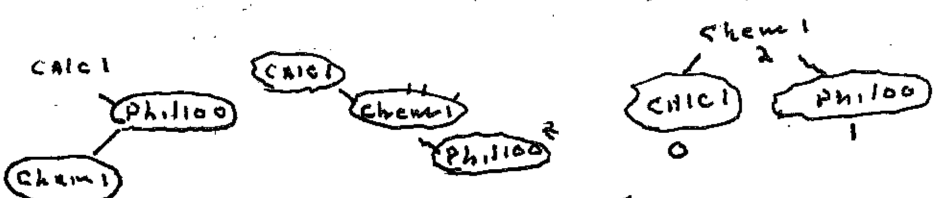
(d) Recall that a modular inverse of an integer b mod m is the integer b^{-1} such that $(b^* b^{-1}) \bmod m=1$.
What is the modular inverse of 3 mod 7? __5__ (4 Points)

(e) In Project 3, we used a Red Black Tree to check for new course names or existing course names – assigning small integers counting from 0.

Draw what the Red Black Tree would look like after the following course names are read from the input. The tree must show the course name and the value assigned to each course.

Draw RED nodes as circled and black nodes as un-circled. If you show each step clearly, you will receive partial credit. (4 Points)

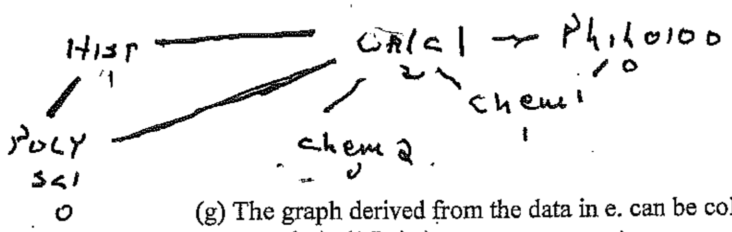
Smith, Amy Calc1, Philo100, Chem1
Bell,Amy Calc1, Chem2
Obama,Barak Calc1, History, PolySci



STOP GRADING AT THIS TREE

(f) In Project 3, we built a graph from data such as in part e. Draw the graph that would result from reading the data in part e. (3 Points) This is not the Red Black Tree.

check Polysci, chicl, history



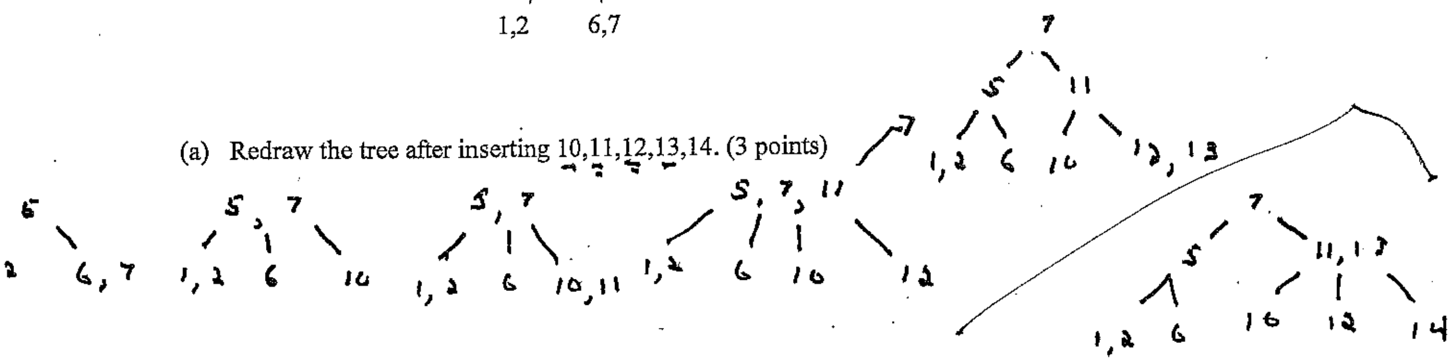
(g) The graph derived from the data in e. can be colored with three colors. (Circle True, or False). (1 Point)

Balanced Trees (15 points)

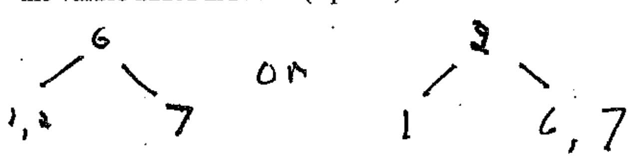
10. Consider the following B-Tree with a minimum of 1 and a maximum of 2.

```
      5
    /   \
  1,2   6,7
```

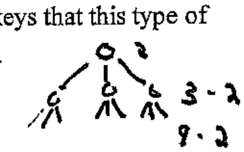
(a) Redraw the tree after inserting 10,11,12,13,14. (3 points)



(b) Delete the value 5 from the B-Tree shown above. Begin work from the original tree, not the tree with the values added in Part a. (2 points)



(c) Consider again the original tree of height of 1. What is the maximum number of keys that this type of tree (min = 1, max=2) could hold with a height of 1? (1 points). 8



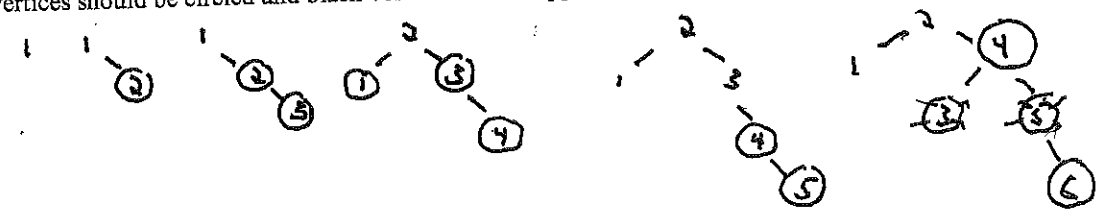
(d) Consider again the original tree of height of 1. What is the maximum number of keys that this type of tree (min = 1, max=2) could hold with a height of 2? (2 points). 26

Key

11. Red Black Trees

(a) Insert the following numbers, one by one, into a Red-Black Tree. Show the tree after each insertion. red vertices should be circled and black vertices should appear without circles. (5 points)

1,2,3,4,5,6



(b) What is the runtime complexity of an inorder traversal of a Red Black Tree ? Use Big Theta notation. (1 Point) $\Theta(N)$

(c) What is the worst-case runtime complexity of a Red Black Tree lookup operation? Use Big Theta notation. (1 point) $\Theta(\lg N)$

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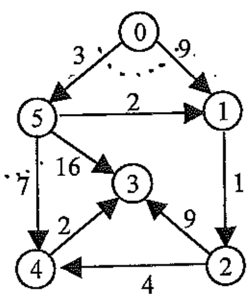
Graph Algorithms(~~25~~ points)

12. (a) What is the shortest path from node 0 to node 3 in the graph immediately below? Your path must be a list of ordered pairs.(1 point) ~~(0,5),(5,1),(1,2),(2,4)~~, (4, 3) OR
(0, 5), (5, 4), (4, 3)

(b) Draw the contents of the distance array for each iteration of Dijkstra's Algorithm as it works on this graph. The initial state is given. **Mark the node to be selected next to the left of the array (note how 0 is marked to the left of the first array.)** Fill in each array cell working downward. That is, complete the first column of arrays before the second column of arrays. (4 Points)



	0	1	2	3	4	5
0	0	9	?	?	?	3
5	0	5	?	19	10	3
1	0	5	6	19	10	3
2	0	5	6	15	10	3
4	0	5	6	12	10	3
3	0	5	6	12	16	3

(c) Draw an adjacency matrix representation for the graph shown above. (2 points)

	0	1	2	3	4	5
0		9				3
1			1			
2				9	4	
3						
4				2		
5		2		16	7	

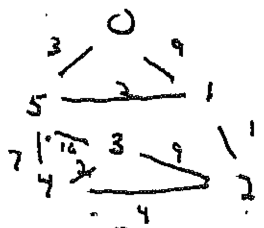
(d) Draw an adjacency set representation of the graph shown immediately above. In this adjacency set representation, each neighbor list is a binary search tree. Be very neat with your drawing. (2 Points)

0 — 1 (9) \ 5 (3)
1 — 2 (1)
2 — 3 (9) \ 4 (4)
3
4 — 3 (2)
5 — 1 (2) \ 3 (16) \ 4 (7)

There are different ways to organize the BST.

(e) The graph shown above is a simple, directed graph. It contains no loops or multiple edges. Suppose we have such a graph with V vertices. What is the maximum number of edges that such a graph can contain? Your answer should be an exact formula. Answers in Big Theta notation don't count. (1 Point) $V^2 - V = V(V-1)$

(f) Redraw the graph in question 14 (a) as an undirected graph. That is, replace each arrow by an undirected edge. The weight of each edge in the undirected graph will be the same as the respective edge weight in the directed graph of 14 (a). (1 Point)



(g) Draw the tree that would be computed by Prim using the graph that you drew in question 14 (f) as input. This is the tree computed by Prim of the undirected, weighted graph you created in 14 (f). (4 points)

1 — 2 — 4 — 3

0 — 5 /

(h) Name an appropriate and fast algorithm to determine whether or not there is a path from one vertex to another in a directed or undirected graph. BFS or, DFS (2 points)

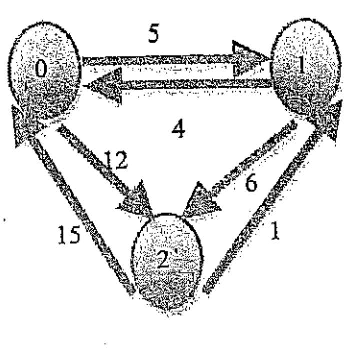
DFS may not work for infinite graphs.

(i) Using the Floyd Warshall algorithm shown below, draw a matrix each time the comment "draw cost matrix" is encountered in the code. Use the graph below for input. You may assume that the original graph is represented in an adjacency matrix c. Also, you must assume that the loops proceed through the graph in numerical order. That is, the first vertex is vertex 0 and the second is vertex 1 and so on. (4 Points)

```
for each vertex u in G do {
  for each vertex v in G do {
    cost[u,v] = c[u,v]
  }
}
```

// draw cost matrix in the first box

```
for each vertex w in G do {
  for each vertex u in G do {
    for each vertex v in G do {
      cost[u,v] = min(cost[u,v],cost[u,w] + cost[w,v]
    } // end for v
  } // end for u
  // draw the cost matrix in the next box
} // end for w
```



Cost

	0	1	2
0		5	12
1	4		6
2	15	1	

	0	1	2
0		5	12
1	4		6
2	15	1	

No change

	0	1	2
0		5	11
1	4		6
2	5	1	

Two changes

	0	1	2
0		5	11
1	4		6
2	5	1	

No change