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Mathematics vs. science

Philosophers have long recognized a distinction between mathematical and scientific reasoning.

Mathematics	Science
about abstract objects	about the world
neither temporal nor spatial	objects in time and space
rational	empirical
deductive	inductive
from axioms to conclusions	from data to laws
certain	fallible
exact, precise	approximate

Mathematics vs. science

This has played out in philosophy in various ways:

- rationalists vs. empiricists in early modern philosophy
- a priori vs. a posteriori in Kant
- analytic / synthetic distinction in twentieth century philosophy

What is at stake:

- ontology: the nature of the objects involved
- epistemology: the proper means of justification
- personal preference

Certainly mathematics and science need each other: "Thoughts without content are empty, and intuitions without concepts are blind."

Two paradigms for Al

"There are two quite different paradigms for AI. Put simply, the logic-inspired paradigm views sequential reasoning as the essence of intelligence and aims to implement reasoning in computers using hand-designed rules of inference that operate on hand-designed symbolic expressions that formalize knowledge. The brain-inspired paradigm views learning representations from data as the essence of intelligence and aims to implement learning by hand-designing or evolving rules for modifying the connection strengths in simulated networks of artificial neurons."

(Yoshua Bengio, Yann Lecun, and Geoffrey Hinton, 2018 Turing Award winners)

A brief history of symbolic Al

Timeline:

- Early twentieth century: axiomatic foundations and decision procedures
- 1940s: first digital computers
- 1956: Dartmouth Summer Research Project on Artificial Intelligence
- 1960s: optimism
- 1970s: disappointment
- 1980s: expert systems, Japan's Fifth Generation Computer Systems project
- 1990s: Al winter (despite Deep Blue)

The symbolic paradigm

Influences:

- cognitive science (symbolic representations, rules of reasoning)
- linguistics and natural language processing (formal grammars, formal semantics)
- automated reasoning (logical rule-based search)
- computer vision (extracting symbolic representations)

The rise of machine learning

Turning points:

- 1990s: big data, statistical methods
- 2011: IBM's Watson wins Jeopardy
- 2012: AlexNet wins ImageNet
- 2016: AlphaGo beats Lee Sedol
- 2022: ChatGPT released

The rise of machine learning

Shifts at Carnegie Mellon:

- Influence of Herbert Simon and Allen Newell in the 1950s and 1960s.
- Logic played a prominent role in Computer Science, Mathematics, Philosophy, Psychology, Linguistics.
- First academic department of Machine Learning in 2006.
- First undergraduate Al major in 2018.

Neural vs. symbolic Al

The web pages for the major include an overview, degree requirements, and a sample curriculum.



The word "logic" does not appear.

Neural vs. symbolic Al

From the New York Times

What is artificial intelligence?

Let's start at the beginning.

The term "artificial intelligence" gets tossed around a lot to describe robots, self-driving cars, facial recognition technology and almost anything else that seems vaguely futuristic.

A group of academics coined the term in the late 1950s as they set out to build a machine that could do anything the human brain could do — skills like reasoning, problem-solving, learning new tasks and communicating using natural language.

Progress was relatively slow until around 2012, when a single idea shifted the entire field.

It was called a **neural network**. That may sound like a computerized brain, but, really, it's a mathematical system that learns skills by finding statistical patterns in enormous amounts of data. By analyzing thousands of cat photos, for instance, it can learn to recognize a cat. Neural networks enable Siri and Alexa to understand what you're saying, identify people and objects in Google Photos and instantly translate dozens of languages.

To some extent, I am conflating

- mathematical reasoning vs. scientific reasoning with
- symbolic methods vs. machine learning.

To be sure, mathematics requires creativity, intuition, experience, big ideas, and deep insights.

But one of the hallmarks of mathematical reasoning is that it employs symbolic reasoning with precise rules and concepts.

Modern AI threatens to replace that.

For those of us who use symbolic methods, the feeling of anxiety is visceral.

Mathematical ideas have supported scientific exploration and practical reasoning for centuries.

Given our limited cognitive abilities, mathematical abstraction has been a valuable tool.

But now that we have neural networks to process the data and tell us what to make of it, maybe the idealized mathematical representations we have been using are less helpful.

Perhaps symbolic methods are no longer important because mathematical reasoning is no longer important: technology has given us something better.

There is a strong aesthetic component to mathematics.

- We do mathematics because it is useful.
- We do mathematics because it is beautiful, and because we enjoy it.

But part of the aesthetic is that mathematics, and precise, rigorous thought, is broadly useful.

And maybe modern technology will change that.

I, personally, think that, in the age of AI, mathematics and symbolic methods are more important than ever.

But that's not something we can take for granted.

Generally, when we ask ChatGPT a question, we want the answer to be

- reliable,
- aligned with our interests,
- likely to help us achieve our goals.

We worry about:

- safety and security
- values and morals.

We want transparency:

- reasons
- explanations
- justification.

This provides one role for mathematics.

But it does go far enough: it presupposes that Al has all the answers, and only needs to explain them to us.

It puts AI, rather than us, at the center of our deliberative processes.

Imagine you are the mayor of a small town, and the town council wants to new bridge.

You say to ChatGPT, "design us a bridge!"



Maybe we want:

- blueprints, with precise lengths and angles
- requirements and tolerances on building materials
- the ability to specify the volume of traffic
- the ability to specify how long it will last
- · an estimate of the cost
- calculations that your engineers can audit and check independently or use to run simulations.

But using mathematics to specify the input, and having mathematics in the output, is not enough.

Important questions:

- What effect will the bridge have on current traffic patterns?
- How will it fare with respect to anticipated growth and changes over the coming decades?
- Should the bridge include paths and walkways to encourage more people to walk and bike, or is it more critical to meet commercial traffic needs?
- How will it affect the environment, and how should we weigh environmental concerns?
- Will the placement of the bridge benefit some residents and harm others?
- What else should you take into account?

It's not just that we need to tell AI what we want and to make sure we get it.

The point is that we often don't know what we want.

Sorting that out requires reasoning and deliberating, individually and with others.

Mathematics provides us with key capacities to reason and deliberate and to come to terms with things like measurements, costs, projections, causes and effects, likelihoods, and uncertainties.

Being rational means not only having goals and values but also deliberating, planning, and coordinating with others to attain them.

Being able to reason about our goals and values presupposes that we can express them to ourselves and to others.

For AI to help us, our interactions have to be mediated by the rich network of concepts and ideas we use to make sense of the world, and mathematics is an essential part of that network.

Formalization and the digitization of mathematics opens up new opportunities for

- verification
- discovery
- exploration
- curation
- communication
- collaboration
- teaching

of mathematics.

Mathematics has been central not only to the scientific method, but to practical decision making in technology, economics, finance, logistics, and public policy.

The AI for mathematics movement is encouraging, since it focuses on

- mathematical concepts, statements, and proofs as outcomes, and
- neurosymbolic methods as a means to achieve them.

Now there are two paths we can follow.

The first involves using AI to support scientific reasoning and decision-making, improving our mathematical models and obtaining a deeper understanding of their properties.

The second involves bypassing mathematics, leaving AI to draw conclusions as it sees fit, and accepting its oracular conclusions.

The first path offers us new means to discover and understand phenomena that would otherwise remain opaque to us, to think and reason better, and to make better decisions.

The second path means turning our back on science, relinquishing agency over our practical decisions, and giving up a vital part of what it means to be human.

Al offers us the choice, but it cannot tell us which path to take.

It's up to us to get it right.