Low-Communication Distributed Optimization via E. Coli Swarm Foraging

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### Differences from Insect Foraging

<table>
<thead>
<tr>
<th>Insect Colonies</th>
<th>Bacteria Swarms</th>
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</thead>
<tbody>
<tr>
<td>agents move food to colony</td>
<td>swarm moves to food</td>
</tr>
<tr>
<td>fixed pheromone trails</td>
<td>diffusing protein signals</td>
</tr>
<tr>
<td>nurses, foragers, queen, etc.</td>
<td>identical cells</td>
</tr>
<tr>
<td>complex navigation abilities</td>
<td>no navigation ability</td>
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</tbody>
</table>
Bacteria Swarm Foraging

- Food source which diffuses with density $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ throughout solution
- Obstacles
- Bacteria swarms (typically 1-4 swarms of 20-50 agents each)
Several nodes each want to maximize the same objective function:

$$\max_{x \in S \subseteq \mathbb{R}^d} f(x).$$

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- $S$ and $f$ typically non-convex
  - Can have small local maxima
- Individual nodes computationally weak
- Nodes can broadcast (small) messages to nearby nodes
Individual Movement (Tumbling)

Each iteration, each agent perturbs its direction based on previous change in food density:

\[
\delta = f(x_t, y_t) - f(x_{t-1}, y_{t-1})
\]

\[
\theta \rightarrow \theta + \varepsilon, \quad \text{where} \quad \varepsilon \sim \mathcal{N}(0, \sigma^2),
\]

\[
\sigma \propto \max\{0, 1 - \delta\}.
\]
Individual Movement (Tumbling)

This works, but very inefficiently:
Basic Swarm Movement (Shklarsh et al., 2011)

On each iteration, each agent combines its (perturbed) velocity with the influence of the swarm

\[ v_{i,t+1} = w_v R_v v_{i,t} + \begin{cases} 
  w_r r_{i,t} & \text{if any neighbors are too close} \\
  w_a a_{i,t} + w_\omega \omega_{i,t} & \text{else}
\end{cases} \]
Avoid collisions and spread out to cover area

\[ r_{i,t} = \sum_{x_{j,t} \in B_{RR}(x_{i})} \frac{x_{j,t} - x_{i,t}}{\|x_{j,t} - x_{i,t}\|}. \]
Basic Swarm Movement (Attraction)

Stay together as a group

\[ a_{i,t} = \sum_{x_{j,t} \in B_{RA}(x_i) \setminus B_{RO}(x_i)} \frac{X_{j,t} - X_{i,t}}{\|X_{j,t} - X_{i,t}\|} \].

\( a_{i,t} \) represents the attraction force acting on agent \( i \) at time \( t \), calculated as the sum of the attraction forces from all other agents \( j \) that are within the attraction range but not within the repulsion range of \( i \).
Basic Swarm Movement (Orientation)

Move similarly to your neighbors

\[
\omega_{i,t} = \sum_{x_j,t \in B_{RO}(x_i)} \frac{V_{j,t}}{\|v_{j,t}\|}.
\]

Accelerates swarm when the correct direction is clear
Helps "smooth" interactions by preventing collisions.
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Basic Swarm Movement (Shklarsh et al.)

Again,

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- Messages can be continuous (e.g., floats)
  - Real bacteria send protein signals of only a few bits
- Receiver’s measurements can be arbitrarily large
  - Real bacteria distinguish only a few levels
Introduce a thresholding discretization function:

- For $T > 0$, $L \in \mathbb{N}$, $\|D_{L,T}(x)\| = \min\{ T, \left\lfloor \frac{L \|x\|}{L} \right\rfloor \}$.  
- Approximate vectors by cardinal vectors to discretize direction.
The Basic Swarm Movement model makes unrealistic assumptions about how bacteria communicate orientation and attraction (repulsion is ok).

- Agents can identify message senders (dedicated channels)
  - Requires $\log(n)$ extra bits per message
  - Swarm can be dynamic
  - Real bacteria broadcast to their neighbors
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- Agents can identify message senders (dedicated channels)
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  - Swarm can be dynamic
  - Real bacteria broadcast to their neighbors
- Ability to communicate is unaffected by distance
Distance Weighting

- Broadcast messages, but weight communication by distance
- Messages decay exponentially with distance:
  \[ w_a(x) = \exp(-c_a x), \quad w_\omega(x) = \exp(-c_\omega x) \quad (c_\omega > c_a) \]
Efficient Communication Model

- Discretize after weighting:

\[ a_{i,t} = \sum_{j=1}^{n} D_{L,T} \left( w_a(\|x_j - x_i\|) \left( \frac{x_j - x_i}{\|x_j - x_i\|} \right) \right) \]

\[ \omega_{i,t} = \sum_{j=1}^{n} D_{L,T} \left( w_a(\|v_j,t\|) \left( \frac{v_j}{\|v_j\|} \right) \right) \]

Recall

\[ v_{i,t+1} = w_v v_{i,t} + \begin{cases} w_r r_{i,t} & \text{if any neighbors are too close} \\ w_a a_{i,t} + w_\omega \omega_{i,t} & \text{else} \end{cases} \]
Experimental Results

Path Length
Help if you’re making progress, get help if you’re stuck
- weight current velocity based on performance

Modified model:

\[ v_t = w(\delta) \cdot v_{t-1} + (1 - w(\delta))u, \]

where \( w \) is increases with \( \delta = f(x_t, y_t) - f(x_{t-1}, y_{t-1}). \)
## Silent Agents

- Broadcasting messages takes energy
- Many messages are redundant
- Under scarce resources, may not want to help competition
Silent Agents

- broadcasting messages takes energy
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Modified model: For some $p_s \in [0, 1]$, each agent is silent with probability $p_s$. 
Experimental Results: Silent Agents

Very few agents actually need to communicate!
Summary

- Primitive bacteria solve computationally challenging problems collectively.
- Swarm communication is helpful even under highly restricted communication:
  - Agents need only broadcast a few bits.
  - Signals only need to travel short distances.
  - Only some agents need to communicate.
Future Work

- Consider competition (finite food sources)
- Multiple food sources/mixed objectives
  - Agents can have different preferences
- Compare to biological model
  - Can identify genes responsible for communication?
  - How is orientation really communicated?
- Theory
  - Convergence rates
  - Lower bounds
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<th>Old Model</th>
<th>New Model</th>
<th>Extensions</th>
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Adaptive Listening
Silent Agents

Thanks!

Simulation code is available on GitHub.